13-1 Areas of Parallelograms and Triangles

TEKS FOCUS

TEKS (11)(B) Determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure.

TEKS (1)(C) Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and **number sense** as appropriate, to solve problems.

Additional TEKS (1)(F), (11)(A)

VOCABULARY

- **Altitude of a parallelogram** – An altitude of a parallelogram is any segment perpendicular to the line containing the base, drawn from the side opposite the base.
- **Base of a parallelogram** – A base of a parallelogram is any one of the parallelogram's sides.
- **Base of a triangle** – A base of a triangle is any one of the triangle's sides.
- **Composite figure** – A composite figure is a combination of two or more figures.
- **Height of a parallelogram** – The height of a parallelogram is the length of an altitude of the parallelogram.
- **Height of a triangle** – The height of a triangle is the length of the altitude to the line containing that base.

- **Number sense** – the understanding of what numbers mean and how they are related

ESSENTIAL UNDERSTANDING

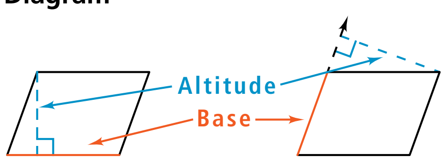
You can find the area of a parallelogram or a triangle when you know the lengths of its base and its height.

take note Key Concept Parts of a Parallelogram

Term Description

A **base of a parallelogram** can be any one of its sides. The corresponding **altitude** is a segment perpendicular to the line containing that base, drawn from the side opposite the base. The **height** is the length of an altitude.

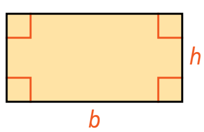
Diagram



take note Key Concept Area of a Rectangle

The area of a rectangle is the product of its base and height.

$A = bh$

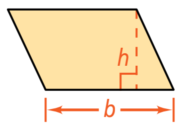


Key Concept Area of a Parallelogram

take note

The area of a parallelogram is the product of a base and the corresponding height.

$A = bh$

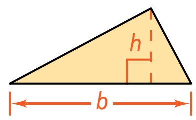


Key Concept Area of a Triangle

take note

The area of a triangle is half the product of a base and the corresponding height.

$A = \frac{1}{2}bh$



Postulate 13-1 Area Addition Postulate

take note

The area of a region is equal to the sum of the areas of its nonoverlapping parts.

Problem 1

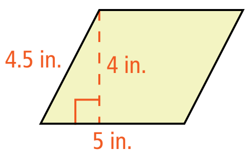
Finding the Area of a Parallelogram

What is the area of each parallelogram?

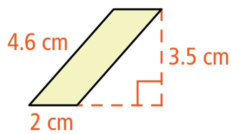
Think

Why aren't the sides of the parallelogram considered altitudes? Altitudes must be perpendicular to the bases. Unless the parallelogram is also a rectangle, the sides are not perpendicular to the bases.

A



B



You are given each height. Choose the corresponding side to use as the base.

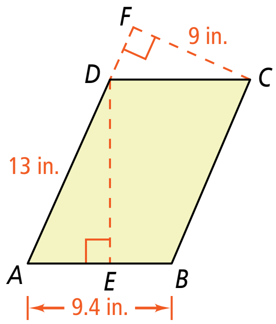
A		B
$A = bh$		$A = bh$
$= 5(4)$	Substitute for b and h.	$= 2(3.5)$
$= 20$		$= 7$
The area is 20 in.^{2}.		The area is 7 cm^2.

Problem 2

TEKS Process Standard (1)(F)

Finding a Missing Dimension

For $\square ABCD$, what is DE to the nearest tenth?



Think

What does $\overline{CF}$ represent?
$\overline{CF}$ is an altitude of the parallelogram when $\overline{AD}$ and $\overline{BC}$ are used as bases.

First, find the area of $\square ABCD$. Then use the area formula a second time to find DE.

$A = bh$

$= 13(9) = 117$ Use base AD and height CF.

The area of $\square ABCD$ is 117 in.^{2}.

$A = bh$

$117 = 9.4(DE)$ Use base AB and height DE.

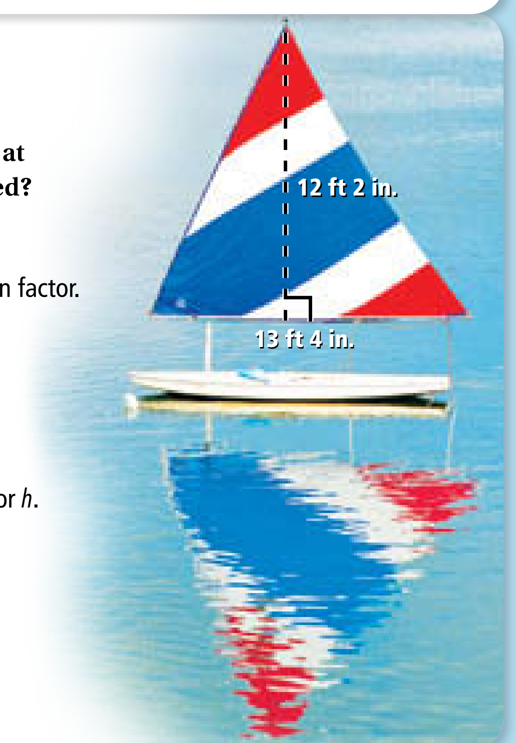
$DE = \frac{117}{9.4} \approx 12.4$

DE is about 12.4 in.

Problem 3

Finding the Area of a Triangle

Sailing **You want to make a triangular sail like the one at the right. How many square feet of material do you need?**



Plan

Why do you need to convert the base and the height into inches?
You must convert them both because you can only multiply measurements with like units.

Step 1 Convert the dimensions of the sail to inches.

$\left(12 \text{ ft} \cdot \frac{12 \text{ in.}}{1 \text{ ft}}\right) + 2 \text{ in.} = 146 \text{ in.}$ Use a conversion factor.

$\left(13 \text{ ft} \cdot \frac{12 \text{ in.}}{1 \text{ ft}}\right) + 4 \text{ in.} = 160 \text{ in.}$

Step 2 Find the area of the triangle.

$A = \frac{1}{2}bh$

$= \frac{1}{2}(160)(146)$ Substitute 160 for b and 146 for h.

$= 11{,}680$ Simplify.

Step 3 Convert 11,680 in.^{2} to square feet.

$11{,}680 \text{ in.}^2 \cdot \frac{1 \text{ ft}}{12 \text{ in.}} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} = 81\frac{1}{9} \text{ ft}^2$

You need $81\frac{1}{9}$ ft^2 of material.

Problem 4

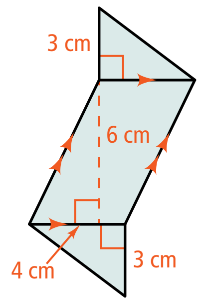
TEKS Process Standard (1)(C)

Finding the Area of a Composite Figure

Select a technique that will help you find the area of the composite figure below. Then find the area of the figure.

Think

Could you divide the composite figure differently and still use the Area Addition Postulate?
Yes, but you might not be able to use mental math to do the calculations.



You can use mental math for this problem, since the calculations are easy enough to do in your head.

The area of each triangle is $\frac{1}{2}(3)(4) = 6$.

The area of the parallelogram is $(6)(4) = 24$.

To find the area of the entire figure, add the areas of the two triangles and the parallelogram.

$$6 + 6 + 24 = 36$$

The area of the composite figure is 36 cm^2.

PRACTICE and APPLICATION EXERCISES

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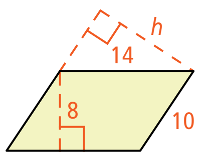
Find the value of h for each parallelogram.

1.



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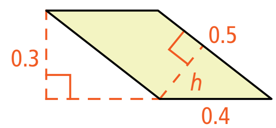
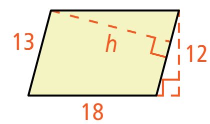
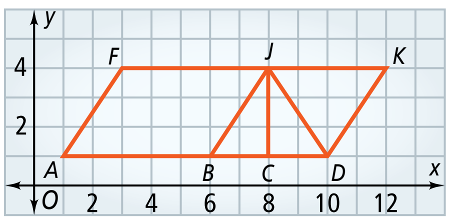


3.



Find the area of each figure.



4. ▱$ABJF$

5. △BDJ

6. △DKJ

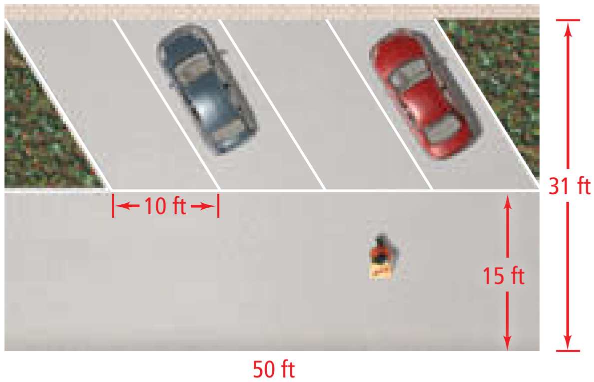
7. ▱$BDKJ$

8. ▱$ADKF$

9. △BCJ

10. trapezoid $ADJF$

11. **Apply Mathematics (1)(A)** A bakery has a 50 ft-by-31 ft parking lot. The four parking spaces are congruent parallelograms, the driving region is a rectangle, and the two areas for flowers are congruent triangles.

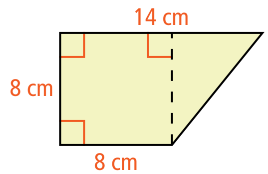


   a. Find the area of the paved surface by adding the areas of the driving region and the four parking spaces.

   b. Describe another method for finding the area of the paved surface.

   c. Use your method from part (b) to find the area. Then compare answers from parts (a) and (b) to check your work.

12. What is the area of the figure at the right?



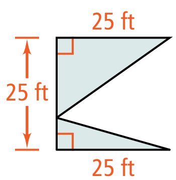
   A. 64 cm^2
   B. 88 cm^2
   C. 96 cm^2
   D. 112 cm^2

13. The area of a parallelogram is 24 $in.^2$, and the height is 6 in. Find the length of the corresponding base.

14. A right isosceles triangle has area 98 cm^2. Find the length of each leg.

15. **Analyze Mathematical Relationships (1)(F)** The area of a triangle is 108 $in.^2$. A base and corresponding height are in the ratio 3 : 2. Find the length of the base and the corresponding height.

Find the area of each figure.

16.


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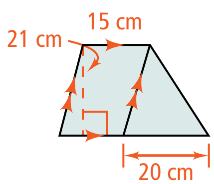

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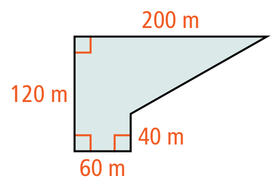
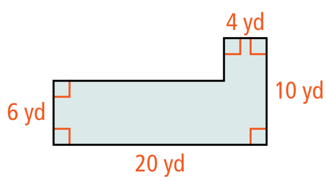


Select Techniques to Solve Problems (1)(C) **Select a technique (such as mental math, estimation, or number sense) to find the area of the composite figure. Then find the area.**

19.


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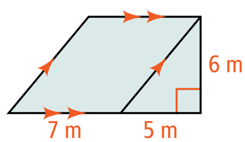


For Exercises 21 and 22, (a) graph the lines and (b) find the area of the triangle enclosed by the lines.

21. $y = -\frac{1}{2}x + 3, y = 0, x = -2$

22. $y = \frac{3}{4}x - 2, y = -2, x = 4$

Find the area of a polygon with the given vertices.

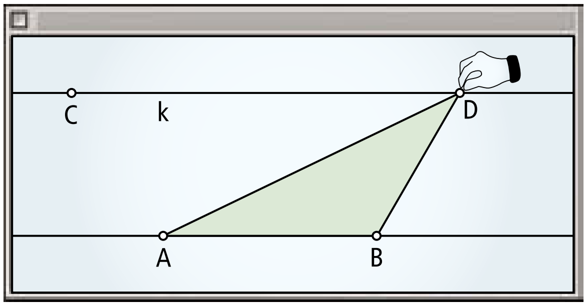
23. $E(1, 1), F(4, 5), G(11, 5), H(8, 1)$

24. $A(3, 9), B(8, 9), C(2, -3), D(-3, -3)$

25. $D(0, 0), E(2, 4), F(6, 4), G(6, 0)$

26. $K(-7, -2), L(-7, 6), M(1, 6), N(7, -2)$

27. **Explain Mathematical Ideas (1)(G)** Ki used geometry software to make the figure at the right. She constructed $\overleftrightarrow{AB}$ and a point C not on $\overleftrightarrow{AB}$. Then she constructed line k parallel to $\overleftrightarrow{AB}$ through point C. Next, Ki constructed point D on line k as well as $\overline{AD}$ and $\overline{BD}$. She dragged point D along line k to manipulate $\triangle ABD$. How does the area of $\triangle ABD$ change? Explain.



The Greek mathematician Heron is most famous for this formula for the area of a triangle in terms of the lengths of its sides a, b, and c.

$$A = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{1}{2}(a+b+c)$$

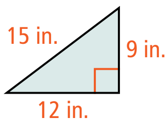
Use Heron's Formula and a calculator to find the area of each triangle. Round your answer to the nearest whole number.

28. $a = 8$ in., $b = 9$ in., $c = 10$ in.

29. $a = 15$ m, $b = 17$ m, $c = 21$ m

30. **a.** Use Heron's Formula to find the area of this triangle.

b. Verify your answer to part (a) by using the formula $A = \frac{1}{2}bh$.



TEXAS Test Practice

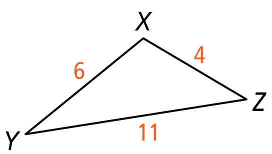
31. The lengths of the sides of a right triangle are 10 in., 24 in., and 26 in. What is the area of the triangle?

A. 116 in.^{2} **B.** 120 in.^{2} **C.** 130 in.^{2} **D.** 156 in.^{2}

32. In quadrilateral $ABCD$, $AB \cong BC \cong CD \cong DA$. Which type of quadrilateral could $ABCD$ never be classified as?

F. square **G.** rectangle **H.** rhombus **J.** kite

33. Are the side lengths of $\triangle XYZ$ possible? Explain.



13-2 Areas of Trapezoids, Rhombuses, and Kites

TEKS FOCUS

TEKS (11)(B) Determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure.

TEKS (1)(B) Use a problem–solving model that incorporates analyzing given information, **formulating** a plan or **strategy**, determining a solution, justifying the solution, and evaluating the problem–solving process and the **reasonableness** of the solution.

Additional TEKS (1)(A), (1)(F), (6)(D), (9)(B)

VOCABULARY

- **Height of a trapezoid** – The height of a trapezoid is the perpendicular distance between the bases.

- **Formulate** – create with careful effort and purpose. You can formulate a plan or strategy to solve a problem.
- **Strategy** – a plan or method for solving a problem
- **Reasonableness** – the quality of being within the realm of common sense or sound reasoning. The reasonableness of a solution is whether or not the solution makes sense.

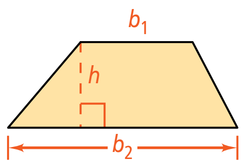
ESSENTIAL UNDERSTANDING

- You can find the area of a trapezoid when you know its height and the lengths of its bases.
- You can find the area of a rhombus or a kite when you know the lengths of its diagonals.

take note — Key Concept Area of a Trapezoid

The area of a trapezoid is half the product of the height and the sum of the bases.

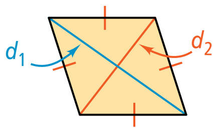
$$A = \frac{1}{2}h(b_1 + b_2)$$



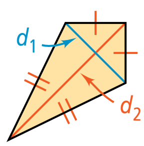
take note — Key Concept Area of a Rhombus or a Kite

The area of a rhombus or a kite is half the product of the lengths of its diagonals.

$$A = \frac{1}{2}d_1 d_2$$



Rhombus



Kite

Problem 1

TEKS Process Standard (1)(A)

Area of a Trapezoid

Plan

Which borders of Nevada can you use as the bases of a trapezoid?
The two parallel sides of Nevada form the bases of a trapezoid.

Geography **What is the approximate area of Nevada?**

$A = \frac{1}{2}h(b_1 + b_2)$ Use the formula for area of a trapezoid.

$= \frac{1}{2}(309)(205 + 511)$ Substitute 309 for h, 205 for b_1, and 511 for b_2.

$= 110{,}622$ Simplify.

The area of Nevada is about 110,600 mi^2.

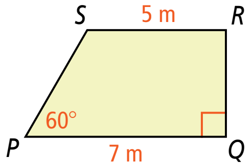
Problem 2

Finding Area Using a Right Triangle

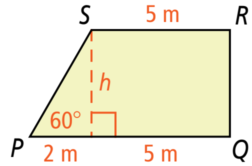
Think

How are the sides related in a 30°-60°-90° triangle?
The length of the hypotenuse is 2 times the length of the shorter leg, and the longer leg is $\sqrt{3}$ times the length of the shorter leg.

What is the area of trapezoid *PQRS*?



You can draw an altitude that divides the trapezoid into a rectangle and a 30°-60°-90° triangle. Since the opposite sides of a rectangle are congruent, the longer base of the trapezoid is divided into segments of lengths 2 m and 5 m.



$h = 2\sqrt{3}$ longer leg = shorter leg • $\sqrt{3}$

$A = \frac{1}{2}h(b_1 + b_2)$ Use the trapezoid area formula.

$= \frac{1}{2}(2\sqrt{3})(7 + 5)$ Substitute $2\sqrt{3}$ for h, 7 for b_1, and 5 for b_2.

$= 12\sqrt{3}$ Simplify.

The area of trapezoid *PQRS* is $12\sqrt{3}$ m^2.

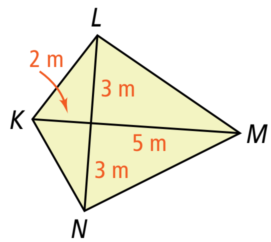
Problem 3

Finding the Area of a Kite

Think

Do you need to know the side lengths of the kite to find its area?
No. You only need the lengths of the diagonals.

What is the area of kite *KLMN*?



Find the lengths of the two diagonals:
$KM = 2 + 5 = 7$ m and $LN = 3 + 3 = 6$ m.

$A = \frac{1}{2}d_1 d_2$ Use the formula for area of a kite.

$= \frac{1}{2}(7)(6)$ Substitute 7 for d_1 and 6 for d_2.

$= 21$ Simplify.

The area of kite *KLMN* is 21 m^2.

Problem 4

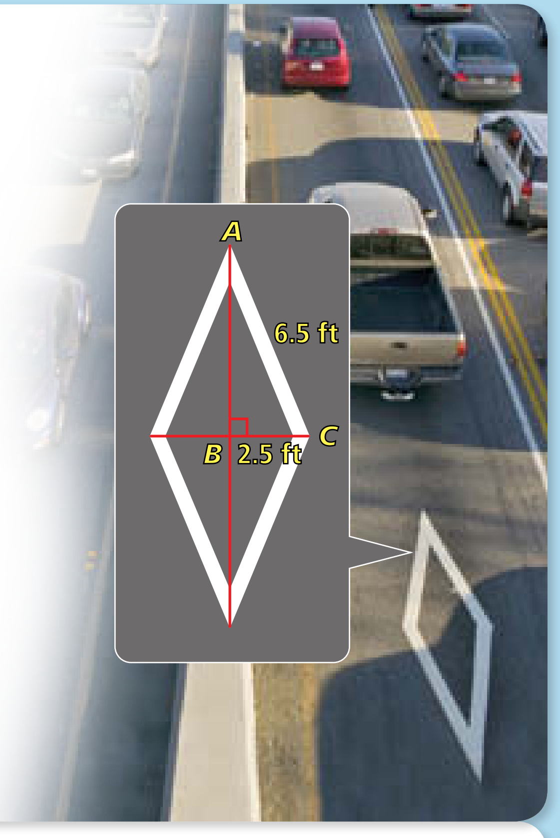
Finding the Area of a Rhombus

Car Pooling **The High Occupancy Vehicle (HOV) lane is marked by a series of "diamonds," or rhombuses painted on the pavement. What is the area of the HOV lane diamond shown at the right?**

Think

How can you find the length of $\overline{AB}$?

$\overline{AB}$ is a leg of right $\triangle ABC$. You can use the Pythagorean Theorem, $a^2 + b^2 = c^2$, to find its length.



$\triangle ABC$ is a right triangle. Using the Pythagorean Theorem, $AB = \sqrt{6.5^2 - 2.5^2} = 6$. Since the diagonals of a rhombus bisect each other, the diagonals of the HOV lane diamond are 5 ft and 12 ft.

$A = \frac{1}{2}d_1 d_2$	Use the formula for area of a rhombus.
$= \frac{1}{2}(5)(12)$	Substitute 5 for d_1 and 12 for d_2.
$= 30$	Simplify.

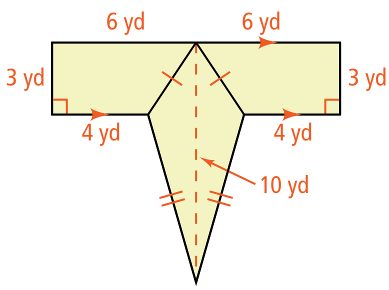
The area of the HOV lane diamond is 30 ft^2.

Problem 5

TEKS Process Standard (1)(B)

Finding the Area of a Composite Figure

Use a problem-solving model to find the area of the figure below.



Analyze the Given Information

Using the definitions of a kite and a trapezoid, you can determine that this figure is composed of a kite and two trapezoids. The diagonal of the kite is 10 yd long, and the two trapezoids both have base lengths of 6 yd and 4 yd, and a height of 3 yd.

Formulate a Plan

To find the area of the composite figure, add the areas of each individual figure.

continued on next page ▶

Problem 5 *continued*

Determine and Justify the Solution

Find the area of each trapezoid.

$$A = \frac{1}{2}h(b_1 + b_2) \quad \text{Use the formula for area of a trapezoid.}$$

$$= \frac{1}{2}(3)(6 + 4) \quad \text{Substitute 3 for } h\text{, 6 for } b_1\text{, and 4 for } b_2.$$

$$= 15 \quad \text{Simplify.}$$

Find the area of the kite. The length of the shorter diagonal is $2(6) - 2(4) = 4$ yd.

$$A = \frac{1}{2}d_1d_2 \quad \text{Use the formula for area of a kite.}$$

$$= \frac{1}{2}(4)(10) \quad \text{Substitute 4 for } d_1 \text{ and 10 for } d_2.$$

$$= 20 \quad \text{Simplify.}$$

Find the total area. The total area is $15 + 15 + 20 = 50$.
So the area of the composite figure is 50 yd^2.

Evaluate the Problem-Solving Process

Think

What should you do if the answer doesn't check?
You should examine the problem-solving process to find mistakes in your reasoning or your calculations.

Check your answer. You can divide the composite figure in a different way, find the area, and compare your answers. You can divide the figure into a rectangle and an isosceles triangle. The base of the rectangle is 12 yd and the height is 3 yd, so its area is 36 yd^2. The triangle has base 4 yd and height 7 yd, so its area is 14 yd^2. $36 + 14 = 50$, so the total area is 50 yd^2. The answer checks.

Since the answer checks, the problem-solving model worked effectively in finding the area of the composite figure.

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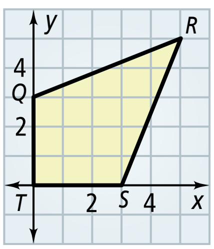
Find the area of quadrilateral *QRST*.

1.



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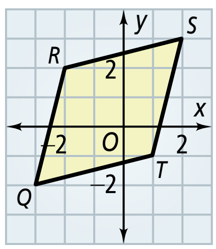

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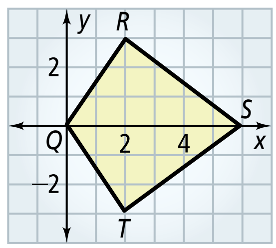
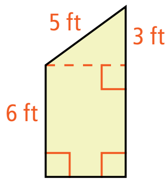


4. The border of Tennessee resembles a trapezoid with bases 340 mi and 440 mi and height 110 mi. Estimate the area of Tennessee by finding the area of the trapezoid.

Find the area of each trapezoid. If your answer is not an integer, leave it in simplest radical form.

5.



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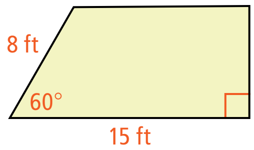

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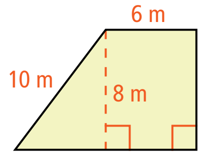
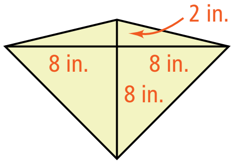


Find the area of each kite.

8.



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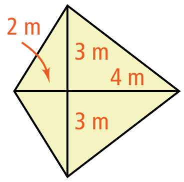

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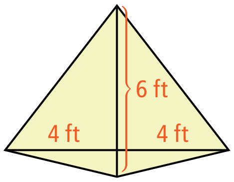
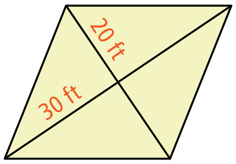


Find the area of each rhombus.

11.



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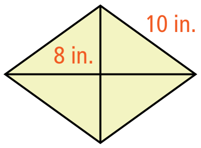

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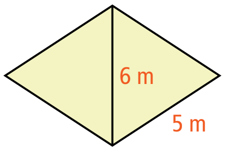
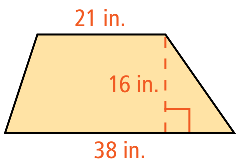


Find the area of each trapezoid.

14.



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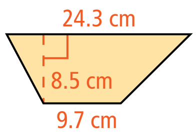

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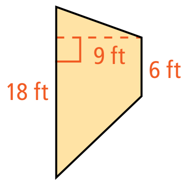
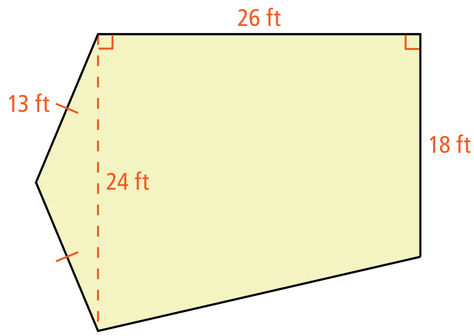


17. Find the area of a trapezoid with bases 12 cm and 18 cm and height 10 cm.

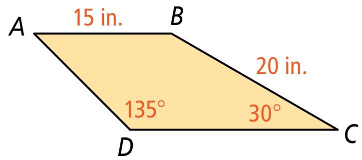
18. Find the area of a trapezoid with bases 2 ft and 3 ft and height $\frac{1}{3}$ ft.

19. **Use a Problem-Solving Model (1)(B)** Find the area of the figure at the right. Use a problem-solving model by

- analyzing the given information
- formulating a plan or strategy
- determining a solution
- justifying the solution
- evaluating the problem-solving process

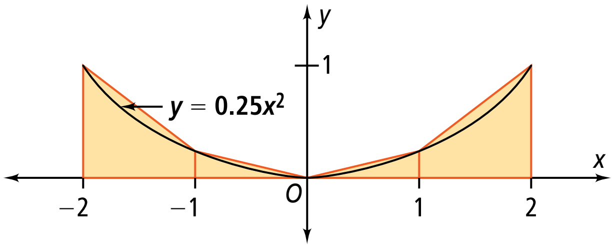


20. In trapezoid $ABCD$ at the right, $\overline{AB} \parallel \overline{DC}$. Find the area of $ABCD$.

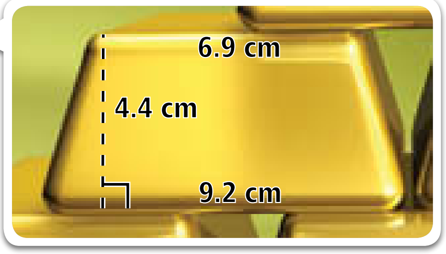


21. **Analyze Mathematical Relationships (1)(F)** One base of a trapezoid is twice the other. The height is the average of the two bases. The area is 324 cm^2. Find the height and the lengths of the bases.

22. **Apply Mathematics (1)(A)** Ty wants to paint one side of the skateboarding ramp he built. The ramp is 4 m wide. Its surface is modeled by the equation $y = 0.25x^2$. Use the trapezoids and triangles shown to estimate the area to be painted.



23. **Apply Mathematics (1)(A)** The end of a gold bar has the shape of a trapezoid with the measurements shown. Find the area of the end.



24. a. **Create Representations to Communicate Mathematical Ideas (1)(E)** Graph the lines $x = 0$, $x = 6$, $y = 0$, and $y = x + 4$.

b. What type of quadrilateral do the lines form?

c. Find the area of the quadrilateral.

TEXAS Test Practice

25. The area of a kite is 120 cm^2. The length of one diagonal is 20 cm. What is the length of the other diagonal?

A. 12 cm

B. 20 cm

C. 24 cm

D. 48 cm

26. $\triangle ABC \sim \triangle XYZ$. $AB = 6$, $BC = 3$, and $CA = 7$. Which of the following are NOT possible dimensions of $\triangle XYZ$?

F. $XY = 3$, $YZ = 1.5$, $ZX = 3.5$

G. $XY = 9$, $YZ = 4.5$, $ZX = 10.5$

H. $XY = 10$, $YZ = 7$, $ZX = 11$

J. $XY = 18$, $YZ = 9$, $ZX = 21$

27. Draw an angle. Construct a congruent angle and its bisector.

13-3 Areas of Regular Polygons

TEKS FOCUS

TEKS (11)(A) Apply the formula for the area of regular polygons to solve problems using appropriate units of measure.

TEKS (1)(F) **Analyze** mathematical relationships to connect and communicate mathematical ideas.

Additional TEKS (1)(D), (9)(B)

VOCABULARY

- **Apothem** – An apothem is the perpendicular distance from the center of a polygon to a side.
- **Center of a regular polygon** – The center of a regular polygon is the center of a circle circumscribed about the polygon.
- **Radius of a regular polygon** – A radius of a regular polygon is the distance from the center of the polygon to a vertex.

- **Analyze** – closely examine objects, ideas, or relationships to learn more about their nature

ESSENTIAL UNDERSTANDING

The area of a regular polygon is related to the distance from the center to a side.

take note

Postulate 13-2

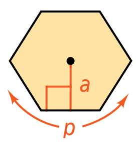
If two figures are congruent, then their areas are equal.

take note

Key Concept Area of a Regular Polygon

The area of a regular polygon is half the product of the apothem and the perimeter.

$$A = \frac{1}{2}ap$$



Problem 1

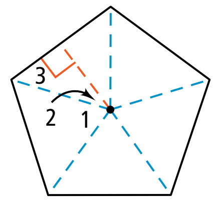
TEKS Process Standard (1)(F)

Finding Angle Measures

Think

How do you know the radii make isosceles triangles?
Since the pentagon is a regular polygon, the radii are congruent. So the triangle made by two adjacent radii and a side of the polygon is an isosceles triangle.

The figure at the right is a regular pentagon with radii and an apothem drawn. What is the measure of each numbered angle?



$m\angle 1 = \frac{360}{5} = 72$ Divide 360 by the number of sides.

$m\angle 2 = \frac{1}{2}m\angle 1$ The apothem bisects the vertex angle of the isosceles triangle formed by the radii.

$= \frac{1}{2}(72) = 36$

$90 + 36 + m\angle 3 = 180$ The sum of the measures of the angles of a triangle is 180.

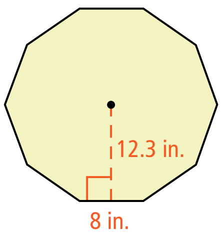
$m\angle 3 = 54$

$m\angle 1 = 72$, $m\angle 2 = 36$, and $m\angle 3 = 54$.

Problem 2

Finding the Area of a Regular Polygon

What is the area of the regular decagon shown below?



Plan

What do you know about the regular decagon?
A decagon has 10 sides, so $n = 10$. From the diagram, you know that the apothem a is 12.3 in., and the side length s is 8 in.

Step 1 Find the perimeter of the regular decagon.

$p = ns$ Use the formula for the perimeter of an n-gon.

$= 10(8)$ Substitute 10 for n and 8 for s.

$= 80$ in.

Step 2 Find the area of the regular decagon.

$A = \frac{1}{2}ap$ Use the formula for the area of a regular polygon.

$= \frac{1}{2}(12.3)(80)$ Substitute 12.3 for a and 80 for p.

$= 492$

The regular decagon has an area of 492 in.^{2}.

Problem 3

TEKS Process Standard (1)(D)

Using Special Triangles to Find Area STEM

Zoology **A honeycomb is made up of regular hexagonal cells. The length of a side of a cell is 3 mm. What is the area of a cell?**

Know
You know the length of a side, which you can use to find the perimeter.

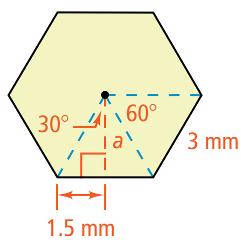
Need
The apothem

Plan
Draw a diagram to help find the apothem. Then use the area formula for a regular polygon.

Step 1 Find the apothem.

The radii form six 60° angles at the center, so you can use a 30°-60°-90° triangle to find the apothem.

$a = 1.5\sqrt{3}$ longer leg $= \sqrt{3} \cdot$ shorter leg



Step 2 Find the perimeter.

$p = ns$ Use the formula for the perimeter of an n-gon.

$= 6(3)$ Substitute 6 for n and 3 for s.

$= 18$ mm

Step 3 Find the area.

$A = \frac{1}{2}ap$ Use the formula for the area of a regular polygon.

$= \frac{1}{2}(1.5\sqrt{3})(18)$ Substitute $1.5\sqrt{3}$ for a and 18 for p.

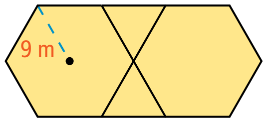
≈ 23.3826859 Use a calculator.

The area is about 23 mm^2.

Problem 4

Finding the Area of a Composite Figure

The figure below is composed of two congruent regular hexagons and two triangles. What is the area of the figure? Round your answer to the nearest square meter.



Step 1 Find the area of one of the regular hexagons.

To find the area of a regular polygon, you need to know the apothem and the perimeter. Use a 30°-60°-90° triangle to find the apothem. Since the hypotenuse is 9 m long, the length of the apothem is $4.5\sqrt{3}$ m. The perimeter of the hexagon is $6 \cdot 9$ m, or 54 m.

$A = \frac{1}{2}ap$	Use the formula for the area of a regular polygon.
$= \frac{1}{2}(4.5\sqrt{3})(54)$	Substitute $4.5\sqrt{3}$ for a and 54 for p.
$= 121.5\sqrt{3}$	Simplify.

Step 2 Find the area of one of the triangles.

The measure of each angle of a regular hexagon is 120. So the measure of an exterior angle is $180 - 120 = 60$. Since two exterior angles of the hexagons make up two angles of the triangle, the measure of all angles of the triangle must be 60. Therefore, it is an equilateral triangle with side length 9 m. Use another 30°-60°-90° triangle to find the height. Since the hypotenuse is 9 m, the height is $4.5\sqrt{3}$ m.

$A = \frac{1}{2}bh$	Use the formula for the area of a triangle.
$= \frac{1}{2}(9)(4.5\sqrt{3})$	Substitute 9 for b and $4.5\sqrt{3}$ for h.
$= 20.25\sqrt{3}$	Simplify.

Step 3 Find the area of the four figures combined.

$$A = 121.5\sqrt{3} + 121.5\sqrt{3} + 20.25\sqrt{3} + 20.25\sqrt{3}$$

$= 283.5\sqrt{3}$	Simplify.
≈ 491.0364039	Use a calculator.

The area of the composite figure is about 491 m^2.

Think

How do you know that the two triangles are congruent?
The triangles are both equilateral, with side lengths of 9 m. So they are congruent.

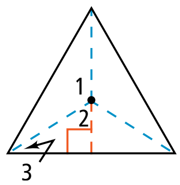
PRACTICE and APPLICATION EXERCISES

Scan page for a Virtual Nerd™ tutorial video.

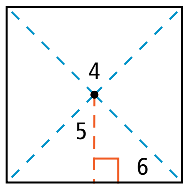
For additional support when completing your homework, go to **PearsonTEXAS.com**.

Each regular polygon has radii and apothem as shown. Find the measure of each numbered angle.

1.



2.



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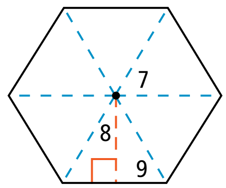


Find the area of each regular polygon with the given apothem *a* and side length *s*.

4. pentagon, $a = 24.3$ cm, $s = 35.3$ cm

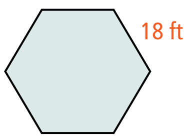
5. octagon, $a = 60.4$ in., $s = 50$ in.

6. nonagon, $a = 27.5$ in., $s = 20$ in.

7. dodecagon, $a = 26.1$ cm, $s = 14$ cm

Find the area of each regular polygon. Round your answer to the nearest tenth.

8.



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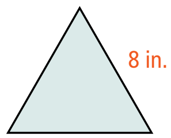

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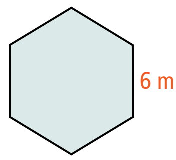
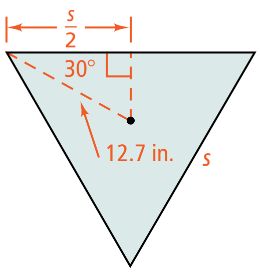
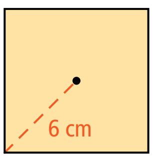


11. Use Multiple Representations to Communicate Mathematical Ideas (1)(D) You are painting a mural of colored equilateral triangles. The radius of each triangle is 12.7 in. What is the area of each triangle to the nearest square inch?



Find the area of each regular polygon with the given radius or apothem. If your answer is not an integer, leave it in simplest radical form.

12.



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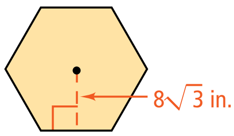

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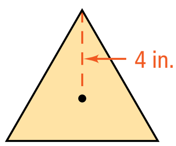


15. **Apply Mathematics (1)(A)** The gazebo in the photo is built in the shape of a regular octagon. Each side is 8 ft long, and the enclosed area is 310.4 ft^2. What is the length of the apothem?

STEM 16. **Apply Mathematics (1)(A)** One of the smallest space satellites ever developed has the shape of a pyramid. Each of the four faces of the pyramid is an equilateral triangle with sides about 13 cm long. What is the area of one equilateral triangular face of the satellite? Round your answer to the nearest whole number.

17. A regular hexagon has perimeter 120 m. Find its area.

18. The area of a regular polygon is 36 $in.^2$. Find the length of a side if the polygon has the given number of sides. Round your answer to the nearest tenth.

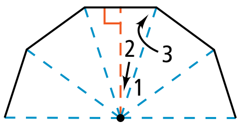
 a. 3

 b. 4

 c. 6

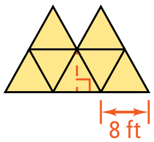
 d. **Select Techniques to Solve Problems (1)(C)** Suppose the polygon is a pentagon. What would you expect the length of a side to be? Explain.

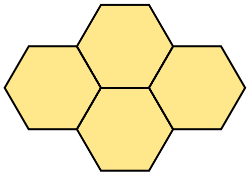
19. A portion of a regular decagon has radii and an apothem drawn. Find the measure of each numbered angle.

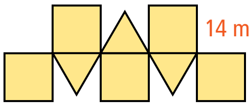


20. **Explain Mathematical Ideas (1)(G)** Explain why the radius of a regular polygon is greater than the apothem.

Find the area of each composite figure. Assume that all parts of figures shown are regular polygons and that figures that are the same shape are congruent. Leave your answer in simplest radical form.

21. 

22. 

23. 

Find the perimeter and area of each regular polygon. Round to the nearest tenth, as necessary.

24. a square with vertices at $(-1, 0)$, $(2, 3)$, $(5, 0)$, and $(2, -3)$

25. a hexagon with two adjacent vertices at $(-2, 1)$ and $(1, 2)$

26. To find the area of an equilateral triangle, you can use the formula $A = \frac{1}{2}bh$ or $A = \frac{1}{2}ap$. A third way to find the area of an equilateral triangle is to use the formula $A = \frac{1}{4}s^2\sqrt{3}$. Verify the formula $A = \frac{1}{4}s^2\sqrt{3}$ in two ways, as follows:

a. Find the area of Figure 1 using the formula $A = \frac{1}{2}bh$.

b. Find the area of Figure 2 using the formula $A = \frac{1}{2}ap$.

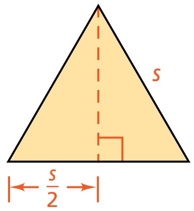


Figure 1

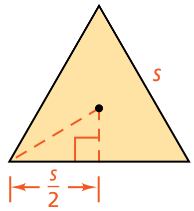
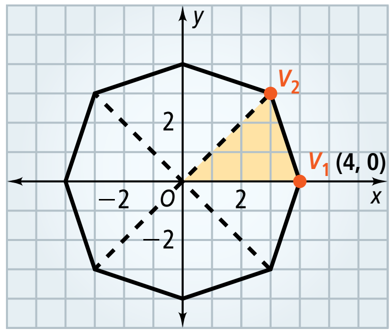


Figure 2

27. For Problem 1, write a proof showing that the apothem
Proof bisects the vertex angle of an isosceles triangle formed by two radii.

28. Prove that the bisectors of the angles of a regular polygon are concurrent and that
Proof they are, in fact, radii of the polygon. (*Hint:* For regular n-gon $ABCDE\ldots$, let P be the intersection of the bisectors of $\angle ABC$ and $\angle BCD$. Show that $\overline{DP}$ must be the bisector of $\angle CDE$.)

29. **Analyze Mathematical Relationships (1)(F)** A regular octagon with center at the origin and radius 4 is graphed in the coordinate plane.



a. Since V_2 lies on the line $y = x$, its x- and y-coordinates are equal. Use the Distance Formula to find the coordinates of V_2 to the nearest tenth.

b. Use the coordinates of V_2 and the formula $A = \frac{1}{2}bh$ to find the area of $\triangle V_1OV_2$ to the nearest tenth.

c. Use your answer to part (b) to find the area of the octagon to the nearest whole number.

TEXAS Test Practice

30. What is the area of a regular pentagon with an apothem of 25.1 mm and a perimeter of 182 mm?

A. 913.6 mm^2 **B.** 2284.1 mm^2 **C.** 3654.6 mm^2 **D.** 4568.2 mm^2

31. What is the most precise name for a regular polygon with four right angles?

F. square **G.** parallelogram **H.** trapezoid **J.** rectangle

32. $\triangle ABC$ has coordinates $A(-2, 4)$, $B(3, 1)$, and $C(0, -2)$. If you reflect $\triangle ABC$ across the x-axis, what are the coordinates of the vertices of the image $\triangle A'B'C'$?

A. $A'(2, 4)$, $B'(-3, 1)$, $C'(0, -2)$

B. $A'(-2, -4)$, $B'(3, -1)$, $C'(0, 2)$

C. $A'(4, -2)$, $B'(1, 3)$, $C'(-2, 0)$

D. $A'(4, 2)$, $B'(1, -3)$, $C'(-2, 0)$

33. An equilateral triangle on a coordinate grid has vertices at (0, 0) and (4, 0). What are the possible locations of the third vertex? Explain.

13-4 Perimeters and Areas of Similar Figures

TEKS FOCUS

TEKS (10)(B) Determine and describe how changes in the linear dimensions of a shape affect its perimeter, area, surface area, or volume, including proportional and non-proportional dimensional change.

TEKS (1)(G) Display, explain, and **justify** mathematical ideas and **arguments** using precise mathematical language in written or oral communication.

Additional TEKS (1)(F), (11)(A)

VOCABULARY

- **Justify** – explain with logical reasoning. You can justify a mathematical argument.
- **Argument** – a set of statements put forth to show the truth or falsehood of a mathematical claim

ESSENTIAL UNDERSTANDING

You can use ratios to compare the perimeters and areas of similar figures.

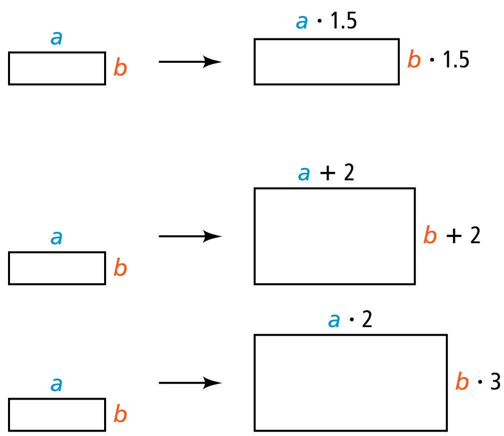
take note — Key Concept Changes in Dimension

A proportional dimensional change multiplies every dimension by the same value. Under a proportional dimensional change, the image of a figure is similar to its preimage.

Under a nonproportional dimensional change, the image of a figure is not similar to its preimage. Two examples of nonproportional dimensional changes are the following:

- Each dimension has the same constant value added to it
- Each dimension is multiplied by a different value

Examples



take note — Key Concept Perimeters and Areas of Similar Figures

If the scale factor of two similar figures is $\frac{a}{b}$, then

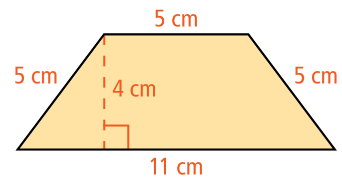
- the ratio of their perimeters is $\frac{a}{b}$
- the ratio of their areas is $\frac{a^2}{b^2}$

Problem 1

TEKS Process Standard (1)(F)

Analyzing Proportional Dimensional Changes

A **How does multiplying each dimension of the isosceles trapezoid by a scale factor of 2 affect its perimeter? How does multiplying each dimension by a scale factor of 3 affect its perimeter?**



Find the perimeter of the trapezoid.

$P = 5 + 11 + 5 + 5 = 26$

Find the dimensions and the perimeter of each scaled trapezoid. Compare the new perimeter to the original perimeter of 26 cm.

Find the new dimensions.

Scale Factor 2

$b_1 = 2 \cdot 5 = 10$
$b_2 = 2 \cdot 11 = 22$
$\text{leg} = 2 \cdot 5 = 10$
$h = 2 \cdot 4 = 8$

Scale Factor 3

$b_1 = 3 \cdot 5 = 15$
$b_2 = 3 \cdot 11 = 33$
$\text{leg} = 3 \cdot 5 = 15$
$h = 3 \cdot 4 = 12$

Find the new perimeter.

$P = 10 + 22 + 10 + 10$
$= 52$

52 cm = 2 • 26 cm

$P = 15 + 33 + 15 + 15$
$= 78$

78 cm = 3 • 26 cm

So, when each dimension is multiplied by 2, the perimeter is multiplied by 2. When each dimension is multiplied by 3, the perimeter is multiplied by 3.

B **How does multiplying each dimension of the isosceles trapezoid in Part A by a scale factor of 2 affect its area? How does multiplying each dimension by a scale factor of 3 affect its area?**

Find the area of the trapezoid.

$A = \frac{1}{2}(4)(5 + 11) = 32$

Use the dimensions you calculated in Part A to find the area of each scaled trapezoid. Compare the new area to the original area of 32 cm^2.

Scale Factor 2

$A = \frac{1}{2}h(b_1 + b_2)$
$= \frac{1}{2}(8)(10 + 22)$
$= 128$

128 cm^2 = 4 • 32 cm^2

Scale Factor 3

$A = \frac{1}{2}h(b_1 + b_2)$
$= \frac{1}{2}(12)(15 + 33)$
$= 288$

288 cm^2 = 9 • 32 cm^2

Think

How does the ratio of the areas appear to be related to the scale factors?
Since 2 • 2 = 4 and 3 • 3 = 9, the ratio of the areas appears to be the square of the scale factors.

So, when each dimension is multiplied by 2, the area is multiplied by 4. When each dimension is multiplied by 3, the area is multiplied by 9.

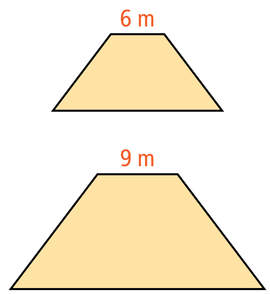
Problem 2

Finding Ratios in Similar Figures

Plan

How do you find the scale factor?
Write the ratio of the lengths of two corresponding sides.

The trapezoids at the right are similar. The ratio of the lengths of corresponding sides is $\frac{6}{9}$, or $\frac{2}{3}$.



A What is the ratio (smaller to larger) of the perimeters?

The ratio of the perimeters is the same as the ratio of corresponding sides, which is $\frac{2}{3}$.

B What is the ratio (smaller to larger) of the areas?

The ratio of the areas is the square of the ratio of corresponding sides, which is $\frac{2^2}{3^2}$, or $\frac{4}{9}$.

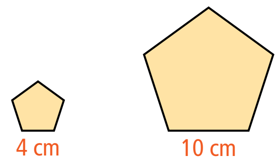
Problem 3

Finding Areas Using Similar Figures

Think

Can you eliminate any answer choices immediately?
Yes. Since the area of the smaller pentagon is 27.5 cm^2, you know that the area of the larger pentagon must be greater than that, so you can eliminate choice A.

Multiple Choice **The area of the smaller regular pentagon is about 27.5 cm^2. What is the best approximation for the area of the larger regular pentagon?**



Ⓐ 11 cm^2 Ⓑ 69 cm^2 Ⓒ 172 cm^2 Ⓓ 275 cm^2

Regular pentagons are similar because all angles measure 108 and all sides in each pentagon are congruent. Here the ratio of corresponding side lengths is $\frac{4}{10}$, or $\frac{2}{5}$. The ratio of the areas is $\frac{2^2}{5^2}$, or $\frac{4}{25}$.

$\frac{4}{25} = \frac{27.5}{A}$ Write a proportion using the ratio of the areas.

$4A = 687.5$ Cross Products Property

$A = \frac{687.5}{4}$ Divide each side by 4.

$A = 171.875$ Simplify.

The area of the larger pentagon is about 172 cm^2. The correct answer is C.

Problem 4

TEKS Process Standard (1)(G)

Applying Area Ratios

Think

Do you need to know the shapes of the two plots of land?
No. As long as the plots are similar, you can compare their areas using their scale factor.

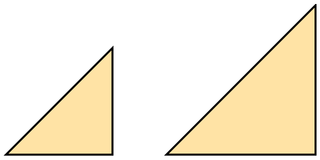
Agriculture **During the summer, a group of high school students cultivated a plot of city land and harvested 13 bushels of vegetables that they donated to a food pantry. Next summer, the city will let them use a larger, similar plot of land. In the new plot, each dimension is 2.5 times the corresponding dimension of the original plot. How many bushels can the students expect to harvest next year?**

The ratio of the dimensions is 2.5 : 1. So the ratio of the areas is $(2.5)^2 : 1^2$, or 6.25 : 1. With 6.25 times as much land next year, the students can expect to harvest 6.25(13), or about 81, bushels.

Problem 5

Finding Perimeter Ratios

The triangles shown below are similar. What is the scale factor? What is the ratio of their perimeters?



Know
The areas of the two similar triangles

Need
The scale factor

Plan
Write a proportion using the ratios of the areas.

$\frac{a^2}{b^2} = \frac{50}{98}$ Use $a^2 : b^2$ for the ratio of the areas.

$\frac{a^2}{b^2} = \frac{25}{49}$ Simplify.

$\frac{a}{b} = \frac{5}{7}$ Take the positive square root of each side.

The ratio of the perimeters equals the scale factor 5 : 7.

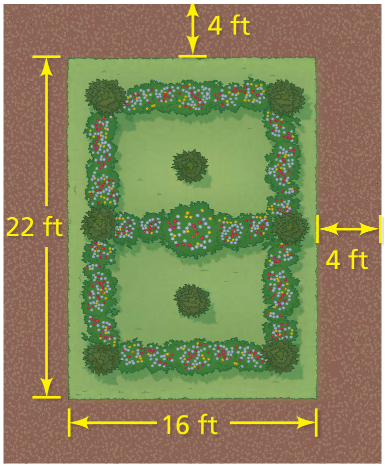
Problem 6

Think

Are the rectangles similar?
No. Since you cannot apply the same scale factor to the lengths of each side of one of the rectangles to get the other, they are not similar.

Analyzing Nonproportional Dimension Changes

The botany club plans to increase the size of a rectangular garden by adding 8 ft to each dimension of the garden.



A **The botany club wants to put fencing around the proposed garden. How many more feet of fencing will the club need to buy for the proposed garden than it would have bought for the current garden?**

Find the perimeter of the current garden.

$$P = 2 \cdot 22 + 2 \cdot 16$$
$$= 76$$

Find the perimeter of the proposed garden.

$$P = 2(22 + 8) + 2(16 + 8)$$
$$= 108$$

Find the difference of the two perimeters.

$$108 - 76 = 32$$

The botany club will need to buy 32 more feet of fencing for the proposed garden.

continued on next page ▶

Problem 6 *continued*

B **The botany club wants to cover the proposed garden with a layer of mulch. How much greater is the area of the proposed garden than the area of the current garden?**

Find the area of the current garden.

$$A = 22 \cdot 16$$
$$= 352$$

Find the area of the proposed garden.

$$A = (22 + 8)(16 + 8)$$
$$= 720$$

Find the difference of the two areas.

$$720 - 352 = 368$$

So the area of the proposed garden is 368 yd^2 greater than the area of the current garden.

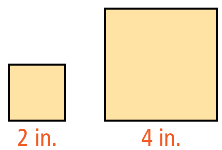
PRACTICE and APPLICATION EXERCISES

Scan page for a Virtual Nerd™ tutorial video.

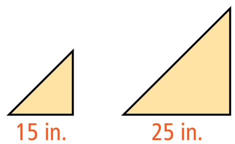
For additional support when completing your homework, go to **PearsonTEXAS.com**.

The figures in each pair are similar. Compare the first figure to the second. Give the ratio of the perimeters and the ratio of the areas.

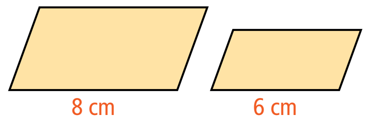
1.



2.



3.



Find the scale factor and the ratio of perimeters for each pair of similar figures.

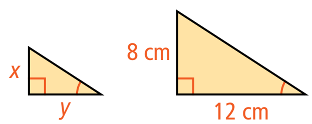
4. two regular octagons with areas 4 ft^2 and 16 ft^2

5. two trapezoids with areas 49 cm^2 and 9 cm^2

6. two equilateral triangles with areas $16\sqrt{3}$ ft^2 and $\sqrt{3}$ ft^2

7. two circles with areas 2π cm^2 and 200π cm^2

Analyze Mathematical Relationships (1)(F) **Find the values of x and y when the smaller triangle shown here has the given area.**

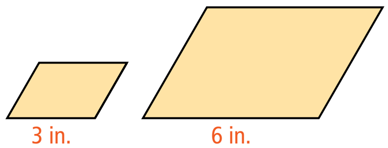


8. 3 cm^2 **9.** 6 cm^2 **10.** 12 cm^2

11. 16 cm^2 **12.** 24 cm^2 **13.** 48 cm^2

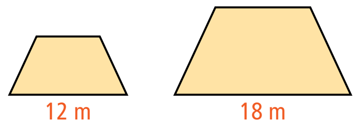
The figures in each pair are similar. The area of one figure is given. Find the area of the other figure to the nearest whole number.

14.



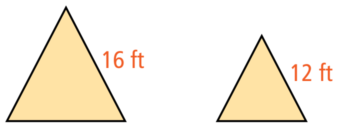
Area of smaller parallelogram = 6 in.^{2}

15.



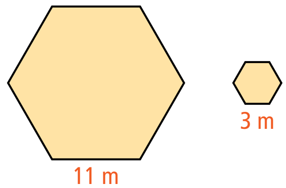
Area of larger trapezoid = 121 m^2

16.



Area of larger triangle = 105 ft^2

17.



Area of smaller hexagon = 23 m^2

18. **Apply Mathematics (1)(A)** An embroidered placemat costs \$3.95. An embroidered tablecloth is similar to the placemat, but four times as long and four times as wide. How much would you expect to pay for the tablecloth?

19. The longer sides of a parallelogram are 5 m. The longer sides of a similar parallelogram are 15 m. The area of the smaller parallelogram is 28 m^2. What is the area of the larger parallelogram?

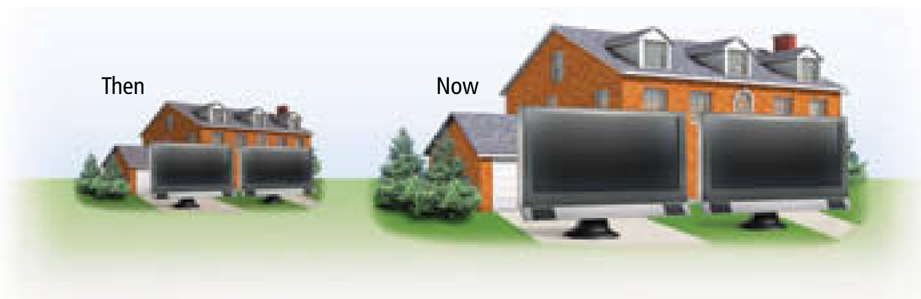
STEM **20.** **Apply Mathematics (1)(A)** For some medical imaging, the scale of the image is 3 : 1. That means that if an image is 3 cm long, the corresponding length on the person's body is 1 cm. Find the actual area of a lesion if its image has area 2.7 cm^2.

21. A rectangular pool and its scale drawing are similar, with a scale factor of 2.5 in. : 11.5 ft. If the dimensions of the drawing are 5.5 in. by 11 in., what is the area of the bottom of the actual pool?

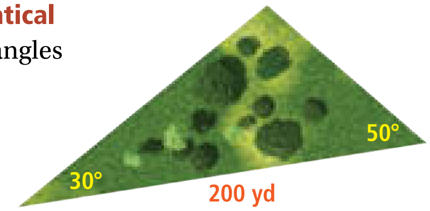
22. A rectangular driveway has a perimeter of 56 feet. If the length is increased by 4 feet, how is the perimeter affected? What is the new perimeter?

23. A postcard has side lengths *s* and *t*. Determine the changes in the area and perimeter of the postcard if the length of *s* is tripled.

24. **Explain Mathematical Ideas (1)(G)** A reporter used the graphic below to show that the number of houses with more than two televisions had doubled in the past few years. Explain why this graphic is misleading.



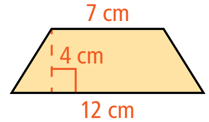
STEM **25. a. Create Representations to Communicate Mathematical Ideas (1)(E)** A surveyor measured one side and two angles of a field, as shown in the diagram. Use a ruler and a protractor to draw a similar triangle.



b. Measure the sides and altitude of your triangle and find its perimeter and area.

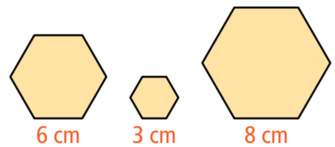
c. Estimate the perimeter and area of the field.

26. Suppose the lengths of both bases of the isosceles trapezoid are halved. Describe how the area of the trapezoid is affected.



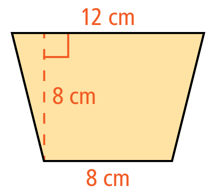
27. a. Find the area of a regular hexagon with sides 2 cm long. Leave your answer in simplest radical form.

b. Use your answer to part (a) and ratios to find the areas of the regular hexagons shown at the right.



28. Justify Mathematical Arguments (1)(G) The enrollment at an elementary school is going to increase from 200 students to 395 students. A parents' group is planning to increase the 100 ft-by-200 ft playground area to a larger area that is 200 ft by 400 ft. What would you tell the parents' group when they ask your opinion about whether the new playground will be large enough?

29. The figure at the right is a scale drawing of a patio. The scale of the drawing is 2 cm = 5 ft. What is the perimeter of the actual patio?



30. A 3 in.-by-5 in. photograph is enlarged by a scale factor of 1 in. : 1.5 ft.

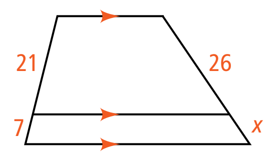
a. Find the perimeter and the area of the enlarged photograph.

b. Suppose the length and width of the enlarged photograph are doubled. Describe how the perimeter and area are affected.

c. Suppose the length (the measure of the longer side) of the enlarged photograph is doubled. Describe how the perimeter and area are affected.

TEXAS Test Practice

31. What is the value of x in the diagram at the right?



32. Two regular hexagons have sides in the ratio 3 : 5. The area of the smaller hexagon is 81 m^2. In square meters, what is the area of the larger hexagon?

33. A trapezoid has base lengths of 9 in. and 4 in. and a height of 3 in. What is the area of the trapezoid in square inches?

34. In quadrilateral $ABCD$, $m\angle A = 62$, $m\angle B = 101$, and $m\angle C = 42$. What is $m\angle D$?