61. Which of the following are equivalent to $\dfrac{\frac{1}{x}}{\frac{3}{y}}$?

a. $\dfrac{1}{x} \div \dfrac{3}{y}$ b. $\dfrac{1}{x} \cdot \dfrac{y}{3}$ c. $\dfrac{1}{x} \div \dfrac{y}{3}$

62. Which of the following are equivalent to $\dfrac{5}{\frac{2}{a}}$?

a. $\dfrac{5}{1} \div \dfrac{2}{a}$ b. $\dfrac{1}{5} \div \dfrac{2}{a}$ c. $\dfrac{5}{1} \cdot \dfrac{2}{a}$

63. In your own words, explain one method for simplifying a complex fraction.

64. Explain your favorite method for simplifying a complex fraction and why.

Simplify.

65. $\dfrac{1}{1 + (1 + x)^{-1}}$

66. $\dfrac{(x + 2)^{-1} + (x - 2)^{-1}}{(x^2 - 4)^{-1}}$

67. $\dfrac{x}{1 - \dfrac{1}{1 + \frac{1}{x}}}$

68. $\dfrac{x}{1 - \dfrac{1}{1 - \frac{1}{x}}}$

69. $\dfrac{\frac{2}{y^2} - \frac{5}{xy} - \frac{3}{x^2}}{\frac{2}{y^2} + \frac{7}{xy} + \frac{3}{x^2}}$

70. $\dfrac{\frac{2}{x^2} - \frac{1}{xy} - \frac{1}{y^2}}{\frac{1}{x^2} - \frac{3}{xy} + \frac{2}{y^2}}$

71. $\dfrac{3(a + 1)^{-1} + 4a^{-2}}{(a^3 + a^2)^{-1}}$

72. $\dfrac{9x^{-1} - 5(x - y)^{-1}}{4(x - y)^{-1}}$

In the study of calculus, the difference quotient $\dfrac{f(a + h) - f(a)}{h}$ *is often found and simplified. Find and simplify this quotient for each function f(x) by following steps* ***a*** *through* ***d.***

a. *Find* $(a + h)$.

b. *Find* $f(a)$.

c. *Use steps* **a** *and* **b** *to find* $\dfrac{f(a + h) - f(a)}{h}$

d. *Simplify the result of step* **c**.

73. $f(x) = \dfrac{1}{x}$

74. $f(x) = \dfrac{5}{x}$

75. $\dfrac{3}{x + 1}$

76. $\dfrac{2}{x^2}$

6.4 Dividing Polynomials: Long Division and Synthetic Division

OBJECTIVES

1. Divide a Polynomial by a Monomial.
2. Divide by a Polynomial.
3. Use Synthetic Division to Divide a Polynomial by a Binomial.
4. Use the Remainder Theorem to Evaluate Polynomials.

OBJECTIVE 1

Dividing a Polynomial by a Monomial

Recall that a rational expression is a quotient of polynomials. An equivalent form of a rational expression can be obtained by performing the indicated division. For example, the rational expression

$$\frac{10x^3 - 5x^2 + 20x}{5x}$$

can be thought of as the polynomial $10x^3 - 5x^2 + 20x$ divided by the monomial $5x$. To perform this division of a polynomial by a monomial (which we do on the next page), recall the following addition fact for fractions with a common denominator.

$$\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}$$

If a, b, and c are monomials, we might read this equation from right to left and gain insight into dividing a polynomial by a monomial.

Dividing a Polynomial by a Monomial

Divide each term in the polynomial by the monomial.

$$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}, \quad \text{where } c \neq 0$$

EXAMPLE 1 Divide $10x^3 - 5x^2 + 20x$ by $5x$.

Solution We divide each term of $10x^3 - 5x^2 + 20x$ by $5x$ and simplify.

$$\frac{10x^3 - 5x^2 + 20x}{5x} = \frac{10x^3}{5x} - \frac{5x^2}{5x} + \frac{20x}{5x} = 2x^2 - x + 4$$

Check: To check, see that (quotient) (divisor) = dividend, or

$$(2x^2 - x + 4)(5x) = 10x^3 - 5x^2 + 20x.$$

□

PRACTICE
1 Divide $18a^3 - 12a^2 + 30a$ by $6a$.

EXAMPLE 2 Divide: $\dfrac{3x^5y^2 - 15x^3y - x^2y - 6x}{x^2y}$.

Solution We divide each term in the numerator by x^2y.

$$\frac{3x^5y^2 - 15x^3y - x^2y - 6x}{x^2y} = \frac{3x^5y^2}{x^2y} - \frac{15x^3y}{x^2y} - \frac{x^2y}{x^2y} - \frac{6x}{x^2y}$$

$$= 3x^3y - 15x - 1 - \frac{6}{xy}$$

□

PRACTICE
2 Divide: $\dfrac{5a^3b^4 - 8a^2b^3 + ab^2 - 8b}{ab^2}$.

OBJECTIVE
2 Dividing by a Polynomial

To divide a polynomial by a polynomial other than a monomial, we use **long division.** Polynomial long division is similar to long division of real numbers. We review long division of real numbers by dividing 7 into 296.

$$\begin{array}{rll} & 42 & \\ \text{Divisor:} & 7\overline{)296} & \\ & -28 & 4(7) = 28. \\ & 16 & \text{Subtract and bring down the next digit in the dividend.} \\ & -14 & 2(7) = 14. \\ & 2 & \text{Subtract. The remainder is 2.} \end{array}$$

The quotient is $42\frac{2}{7}$ $\frac{\text{(remainder)}}{\text{(divisor)}}$.

Check: To check, notice that

$$42(7) + 2 = 296, \text{ the dividend.}$$

This same division process can be applied to polynomials, as shown next.

EXAMPLE 3 Divide $2x^2 - x - 10$ by $x + 2$.

Solution $2x^2 - x - 10$ is the dividend, and $x + 2$ is the divisor.

Step 1. Divide $2x^2$ by x.

$$\begin{array}{r} 2x \\ x + 2\overline{)2x^2 - x - 10} \end{array} \qquad \frac{2x^2}{x} = 2x, \text{ so } 2x \text{ is the first term of the quotient.}$$

(Continued on next page)

Step 2. Multiply $2x(x + 2)$.

$$
\begin{array}{r|l}
 & \phantom{2x^2 - {}}2x \\
\hline
x + 2 & 2x^2 - x - 10 \\
 & \underline{2x^2 - 4x}
\end{array}
$$

$2x(x + 2)$
Like terms are lined up vertically.

Step 3. Subtract $(2x^2 + 4x)$ from $(2x^2 - x - 10)$ by changing the signs of $(2x^2 + 4x)$ and adding.

$$
\begin{array}{r|l}
 & \phantom{2x^2 - {}}2x \\
\hline
x + 2 & 2x^2 - x - 10 \\
 & \underline{\overset{-}{\not+}2x^2 \overset{-}{\not+} 4x} \\
 & -5x
\end{array}
$$

Step 4. Bring down the next term, -10, and start the process over.

$$
\begin{array}{r|l}
 & \phantom{2x^2 - {}}2x \\
\hline
x + 2 & 2x^2 - x - 10 \\
 & \underline{\overset{-}{\not+}2x^2 \overset{-}{\not+} 4x} \quad \downarrow \\
 & -5x - 10
\end{array}
$$

Step 5. Divide $-5x$ by x.

$$
\begin{array}{r|l}
 & \phantom{2x^2 - {}}2x - 5 \\
\hline
x + 2 & 2x^2 - x - 10 \\
 & \underline{\overset{-}{\not+}2x^2 \overset{-}{\not+} 4x} \\
 & -5x - 10
\end{array}
$$

$\frac{-5x}{x} = -5$, so -5 is the second term of the quotient.

Step 6. Multiply $-5(x + 2)$.

$$
\begin{array}{r|l}
 & \phantom{2x^2 - {}}2x - 5 \\
\hline
x + 2 & 2x^2 - x - 10 \\
 & \underline{\overset{-}{\not+}2x^2 \overset{-}{\not+} 4x} \\
 & -5x - 10 \\
 & \underline{-5x - 10}
\end{array}
$$

Multiply: $-5(x + 2)$. Like terms are lined up vertically.

Step 7. Subtract by changing signs of $-5x - 10$ and adding.

$$
\begin{array}{r|l}
 & \phantom{2x^2 - {}}2x - 5 \\
\hline
x + 2 & 2x^2 - x - 10 \\
 & \underline{\overset{-}{\not+}2x^2 \overset{-}{\not+} 4x} \\
 & -5x - 10 \\
 & \underline{\overset{+}{\not-}5x \overset{+}{\not-} 10} \\
 & 0
\end{array}
$$

Subtract.
Remainder

Then $\dfrac{2x^2 - x - 10}{x + 2} = 2x - 5$. There is no remainder.

Check: Check this result by multiplying $2x - 5$ by $x + 2$. Their product is $(2x - 5)(x + 2) = 2x^2 - x - 10$, the dividend. □

PRACTICE 3

Divide $3x^2 + 7x - 6$ by $x + 3$.

EXAMPLE 4 Divide: $(6x^2 - 19x + 12) \div (3x - 5)$.

Solution

$$\begin{array}{r|l} 2x & \\ 3x - 5\,\overline{)\,6x^2 - 19x + 12} & \\ \underline{-6x^2 \overset{+}{\cancel{-}} 10x} \quad \downarrow & \\ -9x + 12 & \end{array}$$

Divide $\frac{6x^2}{3x} = 2x$.

Multiply $2x(3x - 5)$.

Subtract by adding the opposite.

Bring down the next term, $+12$.

$$\begin{array}{r} 2x - 3 \\ 3x - 5\,\overline{)\,6x^2 - 19x + 12} \\ \underline{-6x^2 \overset{+}{\cancel{-}} 10x} \\ -9x + 12 \\ \underline{\overset{+}{\cancel{-}}9x \overset{-}{\cancel{+}} 15} \\ -3 \end{array}$$

Divide $\frac{-9x}{3x} = -3$.

Multiply $-3(3x - 5)$.

Subtract by adding the opposite.

Check: divisor $\cdot$ quotient $+$ remainder

$$(3x - 5) \cdot (2x - 3) + (-3) = 6x^2 - 19x + 15 - 3$$

$$= 6x^2 - 19x + 12 \quad \text{The dividend}$$

The division checks, so

$$\frac{6x^2 - 19x + 12}{3x - 5} = 2x - 3 + \frac{-3}{3x - 5}$$

$$\text{or} \quad 2x - 3 - \frac{3}{3x - 5}$$

Helpful Hint
This fraction is the remainder over the divisor.

□

PRACTICE 4 Divide $(6x^2 - 7x + 8)$ by $(2x - 1)$.

EXAMPLE 5 Divide: $(7x^3 + 16x^2 + 2x - 1) \div (x + 4)$.

Solution

$$\begin{array}{r} 7x^2 - 12x + 50 \\ x + 4\,\overline{)\,7x^3 + 16x^2 + 2x - 1} \\ \underline{-7x^3 \overset{-}{\cancel{+}} 28x^2} \\ -12x^2 + 2x \\ \underline{\overset{+}{\cancel{-}}12x^2 \overset{+}{\cancel{-}} 48x} \\ 50x - 1 \\ \underline{-50x \overset{-}{\cancel{+}} 200} \\ -201 \end{array}$$

Divide $\frac{7x^3}{x} = 7x^2$.

$7x^2(x + 4)$

Subtract. Bring down $2x$.

$\frac{-12x^2}{x} = -12x$, a term of the quotient.

$-12x(x + 4)$ Subtract. Bring down -1.

$\frac{50x}{x} = 50$, a term of the quotient.

$50(x + 4)$. Subtract.

Thus, $\frac{7x^3 + 16x^2 + 2x - 1}{x + 4} = 7x^2 - 12x + 50 + \frac{-201}{x + 4}$ or

$$7x^2 - 12x + 50 - \frac{201}{x + 4}.$$

□

PRACTICE 5 Divide $(5x^3 + 9x^2 - 10x + 30) \div (x + 3)$.

EXAMPLE 6 Divide $3x^4 + 2x^3 - 8x + 6$ by $x^2 - 1$.

Solution Before dividing, we represent any "missing powers" by the product of 0 and the variable raised to the missing power. There is no x^2 term in the dividend, so we include $0x^2$ to represent the missing term. Also, there is no x term in the divisor, so we include $0x$ in the divisor.

$$
\begin{array}{r|l}
 & 3x^2 + 2x + 3 \\
x^2 + 0x - 1 & 3x^4 + 2x^3 + 0x^2 - 8x + 6 \\
 & \underline{-3x^4 \mp 0x^3 \pm 3x^2} \\
 & 2x^3 + 3x^2 - 8x \\
 & \underline{-2x^3 \mp 0x^2 \pm 2x} \\
 & 3x^2 - 6x + 6 \\
 & \underline{-3x^2 \mp 0x \pm 3} \\
 & -6x + 9
\end{array}
$$

- $\frac{3x^4}{x^2} = 3x^2$
- $3x^2(x^2 + 0x - 1)$
- Subtract. Bring down $-8x$.
- $\frac{2x^3}{x^2} = 2x$, a term of the quotient.
- $2x(x^2 + 0x - 1)$
- Subtract. Bring down 6.
- $\frac{3x^2}{x^2} = 3$, a term of the quotient.
- $3(x^2 + 0x - 1)$
- Subtract.

The division process is finished when the degree of the remainder polynomial is less than the degree of the divisor. Thus,

$$\frac{3x^4 + 2x^3 - 8x + 6}{x^2 - 1} = 3x^2 + 2x + 3 + \frac{-6x + 9}{x^2 - 1}$$ □

PRACTICE 6 Divide $2x^4 + 3x^3 - 5x + 2$ by $x^2 + 1$.

EXAMPLE 7 Divide $27x^3 + 8$ by $3x + 2$.

Solution We replace the missing terms in the dividend with $0x^2$ and $0x$.

$$
\begin{array}{r|l}
 & 9x^2 - 6x + 4 \\
3x + 2 & 27x^3 + 0x^2 + 0x + 8 \\
 & \underline{-27x^3 \mp 18x^2} \\
 & -18x^2 + 0x \\
 & \underline{\pm 18x^2 \pm 12x} \\
 & 12x + 8 \\
 & \underline{-12x \mp 8}
\end{array}
$$

- $9x^2(3x + 2)$
- Subtract. Bring down $0x$.
- $-6x(3x + 2)$
- Subtract. Bring down 8.
- $4(3x + 2)$

Thus, $\frac{27x^3 + 8}{3x + 2} = 9x^2 - 6x + 4$. □

PRACTICE 7 Divide $64x^3 - 125$ by $4x - 5$.

> **✓CONCEPT CHECK**
> In a division problem, the divisor is $4x^3 - 5$. The division process can be stopped when which of these possible remainder polynomials is reached?
> **a.** $2x^4 + x^2 - 3$ **b.** $x^3 - 5^2$ **c.** $4x^2 + 25$

OBJECTIVE 3

Using Synthetic Division to Divide a Polynomial by a Binomial

When a polynomial is to be divided by a binomial of the form $x - c$, a shortcut process called **synthetic division** may be used. On the next page, on the left is an example of long division, and on the right, the same example showing the coefficients of the variables only.

Answer to Concept Check: c

$$\begin{array}{r} 2x^2 + 5x + 2 \\ x-3\overline{)2x^3 - x^2 - 13x + 1} \\ \underline{2x^3 - 6x^2} \\ 5x^2 - 13x \\ \underline{5x^2 - 15x} \\ 2x + 1 \\ \underline{2x - 6} \\ 7 \end{array} \qquad \begin{array}{r} 2 \quad 5 \quad 2 \\ 1-3\overline{)2 - 1 - 13 + 1} \\ \underline{2 - 6} \\ 5 - 13 \\ \underline{5 - 15} \\ 2 + 1 \\ \underline{2 - 6} \\ 7 \end{array}$$

Notice that as long as we keep coefficients of powers of x in the same column, we can perform division of polynomials by performing algebraic operations on the coefficients only. This shortcut process of dividing with coefficients only in a special format is called synthetic division. To find $(2x^3 - x^2 - 13x + 1) \div (x - 3)$ by synthetic division, follow the next example.

EXAMPLE 8 Use synthetic division to divide $2x^3 - x^2 - 13x + 1$ by $x - 3$.

Solution To use synthetic division, the divisor must be in the form $x - c$. Since we are dividing by $x - 3$, c is 3. Write down 3 and the coefficients of the dividend.

c

$$\begin{array}{r|rrrr} 3 & 2 & -1 & -13 & 1 \\ & \downarrow & & & \\ \hline & 2 & & & \end{array}$$

Next, draw a line and bring down the first coefficient of the dividend.

$$\begin{array}{r|rrrr} 3 & 2 & -1 & -13 & 1 \\ & & 6 & & \\ \hline & 2 & & & \end{array}$$

Multiply $3 \cdot 2$ and write down the product, 6.

$$\begin{array}{r|rrrr} 3 & 2 & -1 & -13 & 1 \\ & & 6 & & \\ \hline & 2 & 5 & & \end{array}$$

Add $-1 + 6$. Write down the sum, 5.

$$\begin{array}{r|rrrr} 3 & 2 & -1 & -13 & 1 \\ & & 6 & 15 & \\ \hline & 2 & 5 & 2 & \end{array}$$

$3 \cdot 5 = 15$.

$-13 + 15 = 2$.

$$\begin{array}{r|rrrr} 3 & 2 & -1 & -13 & 1 \\ & & 6 & 15 & 6 \\ \hline & 2 & 5 & 2 & 7 \end{array}$$

$3 \cdot 2 = 6$.

$1 + 6 = 7$.

The quotient is found in the bottom row. The numbers 2, 5, and 2 are the coefficients of the quotient polynomial, and the number 7 is the remainder. The degree of the quotient polynomial is one less than the degree of the dividend. In our example, the degree of the dividend is 3, so the degree of the quotient polynomial is 2. As we found when we performed the long division, the quotient is

$$2x^2 + 5x + 2, \quad \text{remainder } 7$$

or

$$2x^2 + 5x + 2 + \frac{7}{x - 3}$$

□

PRACTICE 8 Use synthetic division to divide $4x^3 - 3x^2 + 6x + 5$ by $x - 1$.

When using synthetic division, if there are missing powers of the variable, insert 0s as coefficients.

EXAMPLE 9 Use synthetic division to divide $x^4 - 2x^3 - 11x^2 + 34$ by $x + 2$.

Solution The divisor is $x + 2$, which in the form $x - c$ is $x - (-2)$. Thus, c is -2. There is no x-term in the dividend, so we insert coefficient of 0. The dividend coefficients are 1, −2, −11, 0, and 34.

$$\begin{array}{r|rrrrr} -2 & 1 & -2 & -11 & 0 & 34 \\ & & -2 & 8 & 6 & -12 \\ \hline & 1 & -4 & -3 & 6 & 22 \end{array}$$

The dividend is a fourth-degree polynomial, so the quotient polynomial is a third-degree polynomial. The quotient is $x^3 - 4x^2 - 3x + 6$ with a remainder of 22. Thus,

$$\frac{x^4 - 2x^3 - 11x^2 + 34}{x + 2} = x^3 - 4x^2 - 3x + 6 + \frac{22}{x + 2}$$

□

PRACTICE 9 Use synthetic division to divide $x^4 + 3x^3 - 5x^2 + 12$ by $x + 3$.

✓CONCEPT CHECK

Which division problems are candidates for the synthetic division process?

a. $(3x^2 + 5) \div (x + 4)$ **b.** $(x^3 - x^2 + 2) \div (3x^3 - 2)$

c. $(y^4 + y - 3) \div (x^2 + 1)$ **d.** $x^5 \div (x - 5)$

Helpful Hint

Before dividing by synthetic division, write the dividend in descending order of variable exponents. Any "missing powers" of the variable should be represented by 0 times the variable raised to the missing power.

EXAMPLE 10 If $P(x) = 2x^3 - 4x^2 + 5$,

a. Find $P(2)$ by substitution.

b. Use synthetic division to find the remainder when $P(x)$ is divided by $x - 2$.

Solution

a. $P(x) = 2x^3 - 4x^2 + 5$

$P(2) = 2(2)^3 - 4(2)^2 + 5$

$= 2(8) - 4(4) + 5 = 16 - 16 + 5 = 5$

Thus, $P(2) = 5$.

b. The coefficients of $P(x)$ are 2, −4, 0, and 5. The number 0 is the coefficient of the missing power of x^1. The divisor is $x - 2$, so c is 2.

c →

$$\begin{array}{r|rrrr} 2 & 2 & -4 & 0 & 5 \\ & & 4 & 0 & 0 \\ \hline & 2 & 0 & 0 & 5 \end{array}$$

5 ← remainder

The remainder when $P(x)$ is divided by $x - 2$ is 5. □

Answer to Concept Check:
a and d

PRACTICE
10 If $P(x) = x^3 - 5x - 2$,

a. Find $P(2)$ by substitution.

b. Use synthetic division to find the remainder when $P(x)$ is divided by $x - 2$.

OBJECTIVE
4 Using the Remainder Theorem to Evaluate Polynomials

Notice in the preceding example that $P(2) = 5$ and that the remainder when $P(x)$ is divided by $x - 2$ is 5. This is no accident. This illustrates the **remainder theorem.**

Remainder Theorem

If a polynomial $P(x)$ is divided by $x - c$, then the remainder is $P(c)$.

EXAMPLE 11 Use the remainder theorem and synthetic division to find $P(4)$ if

$$P(x) = 4x^6 - 25x^5 + 35x^4 + 17x^2.$$

Solution To find $P(4)$ by the remainder theorem, we divide $P(x)$ by $x - 4$. The coefficients of $P(x)$ are 4, −25, 35, 0, 17, 0, and 0. Also, c is 4.

c								
4	4	−25	35	0	17	0	0	
		16	−36	−4	−16	4	16	
	4	−9	−1	−4	1	4	16	remainder

Thus, $P(4) = 16$, the remainder.

PRACTICE
11 Use the remainder theorem and synthetic division to find $P(3)$ if $P(x) = 2x^5 - 18x^4 + 90x^2 + 59x$.

Vocabulary, Readiness & Video Check

Martin-Gay Interactive Videos

See Video 6.4

Watch the section lecture video and answer the following questions.

OBJECTIVE 1

1. In the lecture before Example 1, dividing a polynomial by a monomial is compared to adding two fractions. What role does the monomial play in the fraction example?

OBJECTIVE 2

2. From Example 2, how do you know when to stop your long division?

OBJECTIVE 3

3. From Example 3, once you've completed the synthetic division, what does the bottom row of numbers mean? What is the degree of the quotient?

OBJECTIVE 4

4. From Example 4, given a polynomial function $P(x)$, under what circumstances might it be easier/faster to use the remainder theorem to find $P(c)$ rather than substituting the value c for x and then simplifying?

6.4 Exercise Set

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Divide. See Examples 1 and 2.

1. $4a^2 + 8a$ by $2a$
2. $6x^4 - 3x^3$ by $3x^2$
3. $\dfrac{12a^5b^2 + 16a^4b}{4a^4b}$
4. $\dfrac{4x^3y + 12x^2y^2 - 4xy^3}{4xy}$
5. $\dfrac{4x^2y^2 + 6xy^2 - 4y^2}{2x^2y}$
6. $\dfrac{6x^5y + 75x^4y - 24x^3y^2}{3x^4y}$

Divide. See Examples 3 through 7.

7. $(x^2 + 3x + 2) \div (x + 2)$
8. $(y^2 + 7y + 10) \div (y + 5)$
9. $(2x^2 - 6x - 8) \div (x + 1)$
10. $(3x^2 + 19x + 20) \div (x + 5)$
11. $2x^2 + 3x - 2$ by $2x + 4$
12. $6x^2 - 17x - 3$ by $3x - 9$
13. $(4x^3 + 7x^2 + 8x + 20) \div (2x + 4)$
14. $(8x^3 + 18x^2 + 16x + 24) \div (4x + 8)$
15. $(2x^2 + 6x^3 - 18x - 6) \div (3x + 1)$
16. $(4x - 15x^2 + 10x^3 - 6) \div (2x - 3)$
17. $(3x^5 - x^3 + 4x^2 - 12x - 8) \div (x^2 - 2)$
18. $(2x^5 - 6x^4 + x^3 - 4x + 3) \div (x^2 - 3)$
19. $\left(2x^4 + \frac{1}{2}x^3 + x^2 + x\right) \div (x - 2)$
20. $\left(x^4 - \frac{2}{3}x^3 + x\right) \div (x - 3)$

Use synthetic division to divide. See Examples 8 and 9.

21. $\dfrac{x^2 + 3x - 40}{x - 5}$
22. $\dfrac{x^2 - 14x + 24}{x - 2}$
23. $\dfrac{x^2 + 5x - 6}{x + 6}$
24. $\dfrac{x^2 + 12x + 32}{x + 4}$
25. $\dfrac{x^3 - 7x^2 - 13x + 5}{x - 2}$
26. $\dfrac{x^3 + 6x^2 + 4x - 7}{x + 5}$
27. $\dfrac{4x^2 - 9}{x - 2}$
28. $\dfrac{3x^2 - 4}{x - 1}$

MIXED PRACTICE

Divide. See Examples 1–9.

29. $\dfrac{4x^7y^4 + 8xy^2 + 4xy^3}{4xy^3}$
30. $\dfrac{15x^3y - 5x^2y + 10xy^2}{5x^2y}$
31. $(10x^3 - 5x^2 - 12x + 1) \div (2x - 1)$
32. $(20x^3 - 8x^2 + 5x - 5) \div (5x - 2)$
33. $(2x^3 - 6x^2 - 4) \div (x - 4)$
34. $(3x^3 + 4x - 10) \div (x + 2)$
35. $\dfrac{2x^4 - 13x^3 + 16x^2 - 9x + 20}{x - 5}$
36. $\dfrac{3x^4 + 5x^3 - x^2 + x - 2}{x + 2}$
37. $\dfrac{7x^2 - 4x + 12 + 3x^3}{x + 1}$
38. $\dfrac{4x^3 + x^4 - x^2 - 16x - 4}{x - 2}$
39. $\dfrac{3x^3 + 2x^2 - 4x + 1}{x - \frac{1}{3}}$
40. $\dfrac{9y^3 + 9y^2 - y + 2}{y + \frac{2}{3}}$
41. $\dfrac{x^3 - 1}{x - 1}$
42. $\dfrac{y^3 - 8}{y - 2}$
43. $(25xy^2 + 75xyz + 125x^2yz) \div (-5x^2y)$
44. $(x^6y^6 - x^3y^3z + 7x^3y) \div (-7yz^2)$
45. $(9x^5 + 6x^4 - 6x^2 - 4x) \div (3x + 2)$
46. $(5x^4 - 5x^2 + 10x^3 - 10x) \div (5x + 10)$

For the given polynomial $P(x)$ and the given c, use the remainder theorem to find $P(c)$. See Examples 10 and 11.

47. $P(x) = x^3 + 3x^2 - 7x + 4; 1$
48. $P(x) = x^3 + 5x^2 - 4x - 6; 2$
49. $P(x) = 3x^3 - 7x^2 - 2x + 5; -3$
50. $P(x) = 4x^3 + 5x^2 - 6x - 4; -2$
51. $P(x) = 4x^4 + x^2 - 2; -1$
52. $P(x) = x^4 - 3x^2 - 2x + 5; -2$
53. $P(x) = 2x^4 - 3x^2 - 2; \frac{1}{3}$
54. $P(x) = 4x^4 - 2x^3 + x^2 - x - 4; \frac{1}{2}$
55. $P(x) = x^5 + x^4 - x^3 + 3; \frac{1}{2}$
56. $P(x) = x^5 - 2x^3 + 4x^2 - 5x + 6; \frac{2}{3}$

REVIEW AND PREVIEW

Solve each equation for x. See Sections 2.1 and 5.8.

57. $7x + 2 = x - 3$

58. $4 - 2x = 17 - 5x$

59. $x^2 = 4x - 4$

60. $5x^2 + 10x = 15$

61. $\frac{x}{3} - 5 = 13$

62. $\frac{2x}{9} + 1 = \frac{7}{9}$

Factor the following. See Sections 5.5 and 5.7.

63. $x^3 - 1$

64. $8y^3 + 1$

65. $125z^3 + 8$

66. $a^3 - 27$

67. $xy + 2x + 3y + 6$

68. $x^2 - x + xy - y$

69. $x^3 - 9x$

70. $2x^3 - 32x$

CONCEPT EXTENSIONS

Determine whether each division problem is a candidate for the synthetic division process. See the Concept Checks in this section.

71. $(5x^2 - 3x + 2) \div (x + 2)$

72. $(x^4 - 6) \div (x^3 + 3x - 1)$

73. $(x^7 - 2) \div (x^5 + 1)$

74. $(3x^2 + 7x - 1) \div \left(x - \frac{1}{3}\right)$

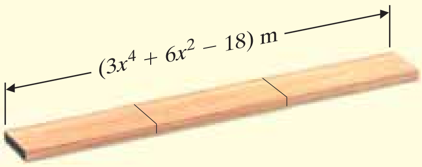
75. In a long division exercise, if the divisor is $9x^3 - 2x$, the division process can be stopped when the degree of the remainder is

a. 1 **b.** 3 **c.** 9 **d.** 2

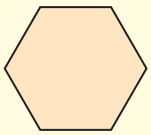
76. In a division exercise, if the divisor is $x - 3$, the division process can be stopped when the degree of the remainder is

a. 1 **b.** 0 **c.** 2 **d.** 3

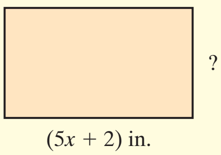
△ **77.** A board of length $(3x^4 + 6x^2 - 18)$ meters is to be cut into three pieces of the same length. Find the length of each piece.



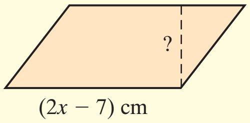
△ **78.** The perimeter of a regular hexagon is given to be $(12x^5 - 48x^3 + 3)$ miles. Find the length of each side.



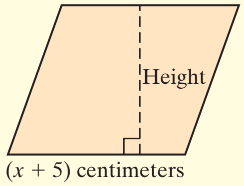
△ **79.** If the area of the rectangle is $(15x^2 - 29x - 14)$ square inches, and its length is $(5x + 2)$ inches, find its width.



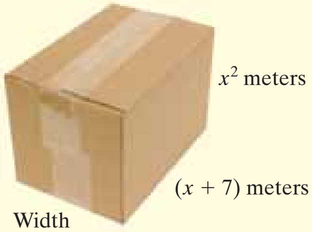
△ **80.** If the area of a parallelogram is $(2x^2 - 17x + 35)$ square centimeters and its base is $(2x - 7)$ centimeters, find its height.



△ **81.** If the area of a parallelogram is $(x^4 - 23x^2 + 9x - 5)$ square centimeters and its base is $(x + 5)$ centimeters, find its height.



△ **82.** If the volume of a box is $(x^4 + 6x^3 - 7x^2)$ cubic meters, its height is x^2 meters, and its length is $(x + 7)$ meters, find its width.



Divide.

83. $\left(x^4 + \frac{2}{3}x^3 + x\right) \div (x - 1)$

84. $\left(2x^3 + \frac{9}{2}x^2 - 4x - 10\right) \div (x + 2)$

85. $\left(3x^4 - x - x^3 + \frac{1}{2}\right) \div (2x - 1)$

86. $\left(2x^4 + \frac{1}{2}x^3 - \frac{1}{4}x^2 + x\right) \div (2x + 1)$

87. $(5x^4 - 2x^2 + 10x^3 - 4x) \div (5x + 10)$

88. $(9x^5 + 6x^4 - 6x^2 - 4x) \div (3x + 2)$

For each given f(x) and g(x), find $\frac{f(x)}{g(x)}$. Also find any x-values that are not in the domain of $\frac{f(x)}{g(x)}$. (Note: Since g(x) is in the denominator, g(x) cannot be 0.)

89. $f(x) = 25x^2 - 5x + 30;\ g(x) = 5x$

90. $f(x) = 12x^4 - 9x^3 + 3x - 1;\ g(x) = 3x$

91. $f(x) = 7x^4 - 3x^2 + 2;\ g(x) = x - 2$

92. $f(x) = 2x^3 - 4x^2 + 1;\ g(x) = x + 3$

93. Try performing the following division without changing the order of the terms. Describe why this makes the process more complicated. Then perform the division again after putting the terms in the dividend in descending order of exponents.

$$\frac{4x^2 - 12x - 12 + 3x^3}{x - 2}$$

94. Explain how to check polynomial long division.

95. Explain an advantage of using the remainder theorem instead of direct substitution.

96. Explain an advantage of using synthetic division instead of long division.

We say that 2 is a factor of 8 because 2 divides 8 evenly, or with a remainder of 0. In the same manner, the polynomial $x - 2$ is a factor of the polynomial $x^3 - 14x^2 + 24x$ because the remainder is 0 when $x^3 - 14x^2 + 24x$ is divided by $x - 2$. Use this information for Exercises 97 and 98.

97. Use synthetic division to show that $x + 3$ is a factor of $x^3 + 3x^2 + 4x + 12$.

98. Use synthetic division to show that $x - 2$ is a factor of $x^3 - 2x^2 - 3x + 6$.

99. If a polynomial is divided by $x - 5$, the quotient is $2x^2 + 5x - 6$ and the remainder is 3. Find the original polynomial.

100. If a polynomial is divided by $x + 3$, the quotient is $x^2 - x + 10$ and the remainder is -2. Find the original polynomial.

101. eBay is the leading online auction house. eBay's annual net profit can be modeled by the polynomial function $P(x) = 0.48x^3 + 2.06x^2 + 141x + 9.71$, where $P(x)$ is net profit in millions of dollars and x is the number of years since 2000. eBay's annual revenue can be modeled by the function $R(x) = 1011x - 288$, where $R(x)$ is revenue in millions of dollars and x is years after 2000. (*Source:* eBay, Inc., annual reports 2000–2010)

a. Given that

$$\text{Net profit margin} = \frac{\text{net profit}}{\text{revenue}},$$

write a function, $m(x)$, that models eBay's net profit margin.

b. Use part (a) to predict eBay's profit margin in 2015. Round to the nearest hundredth.

102. Kraft Foods is a provider of many of the best-known food brands in our supermarkets. Among their well-known brands are Kraft, Oscar Mayer, Maxwell House, and Oreo. Kraft Foods' annual revenues since 2005 can be modeled by the polynomial function $R(x) = 0.06x^3 + 0.02x^2 + 1.67x + 32.33$, where $R(x)$ is revenue in billions of dollars and x is the number of years since 2005. Kraft Foods' net profit can be modeled by the function $P(x) = 0.07x^3 - 0.42x^2 + 0.7x + 2.63$, where $P(x)$ is the net profit in billions of dollars and x is the number of years since 2005. (*Source:* Based on information from Kraft Foods)

a. Suppose that a market analyst has found the model $P(x)$ and another analyst at the same firm has found the model $R(x)$. The analysts have been asked by their manager to work together to find a model for Kraft Foods' profit margin. The analysts know that a company's profit margin is the ratio of its profit to its revenue. Describe how these two analysts could collaborate to find a function $m(x)$ that models Kraft Foods' net profit margin based on the work they have done independently.

b. Without actually finding $m(x)$, give a general description of what you would expect the answer to be.

103. From the remainder theorem, the polynomial $x - c$ is a factor of a polynomial function $P(x)$ if $P(c)$ is what value?

6.5 Solving Equations Containing Rational Expressions

OBJECTIVE

1 Solve Equations Containing Rational Expressions.

OBJECTIVE

1 Solving Equations Containing Rational Expressions

In this section, we solve equations containing rational expressions. Before beginning this section, make sure that you understand the difference between an *equation* and an *expression*. An **equation** contains an equal sign and an **expression** does not.

Equation	*Expression*
$\frac{x}{2} + \frac{x}{6} = \frac{2}{3}$ (↑ equal sign)	$\frac{x}{2} + \frac{x}{6}$

Helpful Hint

The method described here is for equations only. It may *not* be used for performing operations on expressions.

Solving Equations Containing Rational Expressions

To solve *equations* containing rational expressions, first clear the equation of fractions by multiplying both sides of the equation by the LCD of all rational expressions. Then solve as usual.