86. Find the area and perimeter of the trapezoid. \( \text{Hint: The area of a trapezoid is the product of half the height 6}\sqrt{3} \text{ meters and the sum of the bases 2}\sqrt{63} \text{ and 7}\sqrt{7} \text{ meters.} \)

87. a. Add: \( \sqrt{3} + \sqrt{3} \).

b. Multiply: \( \sqrt{3} \cdot \sqrt{3} \).

c. Describe the differences in parts (a) and (b).

88. a. Add: \( 2\sqrt{5} + \sqrt{5} \).

b. Multiply: \( 2\sqrt{5} \cdot \sqrt{5} \).

c. Describe the differences in parts (a) and (b).

89. Multiply: \( (\sqrt{2} + \sqrt{3} - 1)^2 \).

90. Multiply: \( (\sqrt{5} - \sqrt{2} + 1)^2 \).

91. Explain how simplifying \( 2x + 3x \) is similar to simplifying \( 2\sqrt{x} + 3\sqrt{x} \).

92. Explain how multiplying \( (x - 2)(x + 3) \) is similar to multiplying \( (\sqrt{x} - \sqrt{2})(\sqrt{x} + 3) \).

7.5 Rationalizing Denominators and Numerators of Radical Expressions

**OBJECTIVES**

1. Rationalize Denominators.
2. Rationalize Denominators Having Two Terms.
3. Rationalize Numerators.

**OBJECTIVE 1** Rationalizing Denominators of Radical Expressions

Often in mathematics, it is helpful to write a radical expression such as \( \frac{\sqrt{3}}{\sqrt{2}} \) either without a radical in the denominator or without a radical in the numerator. The process of writing this expression as an equivalent expression but without a radical in the denominator is called rationalizing the denominator. To rationalize the denominator of \( \frac{\sqrt{3}}{\sqrt{2}} \), we use the fundamental principle of fractions and multiply the numerator and denominator by \( \sqrt{2} \). Recall that this is the same as multiplying by \( \frac{\sqrt{2}}{\sqrt{2}} \), which simplifies to 1.

\[
\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{6}}{2}
\]

In this section, we continue to assume that variables represent positive real numbers.

**EXAMPLE 1** Rationalize the denominator of each expression.

a. \( \frac{2}{\sqrt{5}} \)

b. \( \frac{2\sqrt{16}}{\sqrt{9x}} \)

c. \( \frac{\sqrt{1}}{\sqrt{2}} \)

**Solution**

a. To rationalize the denominator, we multiply the numerator and denominator by a factor that makes the radicand in the denominator a perfect square.

\[
\frac{2}{\sqrt{5}} = \frac{2 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{2\sqrt{5}}{5}
\]

The denominator is now rationalized.

b. First, we simplify the radicals and then rationalize the denominator.

\[
\frac{2\sqrt{16}}{\sqrt{9x}} = \frac{2(4)}{3\sqrt{x}} = \frac{8}{3\sqrt{x}}
\]

To rationalize the denominator, multiply the numerator and denominator by \( \sqrt{x} \).

Then

\[
\frac{8}{3\sqrt{x}} = \frac{8 \cdot \sqrt{x}}{3\sqrt{x} \cdot \sqrt{x}} = \frac{8\sqrt{x}}{3x}
\]
c. \( \sqrt[3]{\frac{1}{2}} = \frac{\sqrt[3]{1}}{\sqrt[3]{2}} = \frac{1}{\sqrt[3]{2}} \). Now we rationalize the denominator. Since \( \sqrt[3]{2} \) is a cube root, we want to multiply by a value that will make the radicand 2 a perfect cube. If we multiply \( \sqrt[3]{2} \) by \( \sqrt[3]{2^2} \), we get \( \sqrt[3]{2^3} = \sqrt[3]{8} = 2 \).

\[
\frac{1 \cdot \sqrt[3]{2^2}}{\sqrt[3]{2} \cdot \sqrt[3]{2^2}} = \frac{\sqrt[3]{4}}{\sqrt[3]{2^3}} = \frac{\sqrt[3]{4}}{2}
\]

Multiply the numerator and denominator by \( \sqrt[3]{2^2} \) and then simplify.

**Practice 1** Rationalize the denominator of each expression.

a. \( \frac{5}{\sqrt{3}} \)  

b. \( \frac{3\sqrt{25}}{\sqrt{4x}} \)  

c. \( \frac{\sqrt[3]{2}}{9} \)

**Concept Check**

Determine by which number both the numerator and denominator can be multiplied to rationalize the denominator of the radical expression.

a. \( \frac{1}{\sqrt[3]{7}} \)  

b. \( \frac{1}{\sqrt[3]{8}} \)

**Example 2** Rationalize the denominator of \( \frac{\sqrt[3]{7x}}{3y} \).

**Solution**

\[
\frac{\sqrt[3]{7x}}{3y} = \frac{\sqrt[3]{7x}}{\sqrt[3]{3y}} \quad \text{Use the quotient rule. No radical may be simplified further.}
\]

\[
= \frac{\sqrt[3]{7x} \cdot \sqrt[3]{3y}}{\sqrt[3]{3y} \cdot \sqrt[3]{3y}} \quad \text{Multiply numerator and denominator by} \ \sqrt[3]{3y} \ \text{so that the radicand in the denominator is a perfect square.}
\]

\[
= \frac{\sqrt[3]{21xy}}{3y} \quad \text{Use the product rule in the numerator and denominator. Remember that} \ \sqrt[3]{3y} \cdot \sqrt[3]{3y} = 3y.
\]

**Practice 2** Rationalize the denominator of \( \frac{\sqrt[3]{z}}{\sqrt[3]{5y}} \)

**Example 3** Rationalize the denominator of \( \frac{\sqrt[4]{x}}{\sqrt[4]{81y^3}} \).

**Solution**

First, simplify each radical if possible.

\[
\frac{\sqrt[4]{x}}{\sqrt[4]{81y^3}} = \frac{\sqrt[4]{x}}{\sqrt[4]{81y^3}} \cdot \sqrt[4]{y} \quad \text{Use the product rule in the denominator.}
\]

\[
= \frac{\sqrt[4]{x}}{\sqrt[4]{81y^3} \cdot \sqrt[4]{y}} \quad \text{Write} \ \sqrt[4]{81y^3} \ \text{as} \ \sqrt[4]{3y^3}.
\]

\[
= \frac{\sqrt[4]{x} \cdot \sqrt[4]{y}^3}{\sqrt[4]{3y^3} \cdot \sqrt[4]{y}^3} \quad \text{Multiply numerator and denominator by} \ \sqrt[4]{y^3} \ \text{so that the radicand in the denominator is a perfect fourth power.}
\]

\[
= \frac{\sqrt[4]{xy^3}}{\sqrt[4]{3y^3} \cdot \sqrt[4]{y^3}} \quad \text{Use the product rule in the numerator and denominator.}
\]

\[
= \frac{\sqrt[4]{xy^3}}{3y^2} \quad \text{In the denominator,} \ \sqrt[4]{y^3} = y \ \text{and} \ 3y \cdot y = 3y^2.
\]

**Answer to Concept Check:**

a. \( \sqrt[3]{7} \) or \( \sqrt[3]{49} \)  

b. \( \sqrt[3]{2} \)

**Practice 3** Rationalize the denominator of \( \frac{\sqrt[4]{z^3}}{\sqrt[4]{27x^4}} \)


OBJECTIVE 2 Rationalizing Denominators Having Two Terms

Remember the product of the sum and difference of two terms?

\[(a + b)(a - b) = a^2 - b^2\]

These two expressions are called conjugates of each other.

To rationalize a numerator or denominator that is a sum or difference of two terms, we use conjugates. To see how and why this works, let’s rationalize the denominator of the expression \(\frac{5}{\sqrt{3} - 2}\). To do so, we multiply both the numerator and the denominator by \(\sqrt{3} + 2\), the conjugate of the denominator \(\sqrt{3} - 2\), and see what happens.

\[
\frac{5}{\sqrt{3} - 2} = \frac{5(\sqrt{3} + 2)}{\sqrt{3} - 2} \quad \text{(Multiply the sum and difference of two terms: } (a + b)(a - b) = a^2 - b^2. \text{)}
\]

\[
= \frac{5\sqrt{3} + 10}{3 - 4}
\]

\[
= \frac{5\sqrt{3} + 10}{-1}
\]

\[
= -5(\sqrt{3} + 2) \quad \text{or} \quad -5\sqrt{3} - 10
\]

Notice in the denominator that the product of \((\sqrt{3} - 2)\) and its conjugate, \((\sqrt{3} + 2)\), is \(-1\). In general, the product of an expression and its conjugate will contain no radical terms. This is why, when rationalizing a denominator or a numerator containing two terms, we multiply by its conjugate. Examples of conjugates are \(\sqrt{a} - \sqrt{b}\) and \(\sqrt{a} + \sqrt{b}\), \(x + \sqrt{y}\) and \(x - \sqrt{y}\).

EXAMPLE 4 Rationalize each denominator.

a. \(\frac{2}{3\sqrt{2} + 4}\)  
b. \(\frac{\sqrt{6} + 2}{\sqrt{5} - \sqrt{3}}\)  
c. \(\frac{2\sqrt{m}}{3\sqrt{x} + \sqrt{m}}\)

Solution

a. Multiply the numerator and denominator by the conjugate of the denominator, \(3\sqrt{2} + 4\).

\[
\frac{2}{3\sqrt{2} + 4} = \frac{2(3\sqrt{2} - 4)}{(3\sqrt{2} + 4)(3\sqrt{2} - 4)}
\]

\[
= \frac{2(3\sqrt{2} - 4)}{(3\sqrt{2})^2 - 4^2}
\]

\[
= \frac{2(3\sqrt{2} - 4)}{18 - 16}
\]

\[
= \frac{2(3\sqrt{2} - 4)}{2}, \quad \text{or} \quad 3\sqrt{2} - 4
\]

It is often useful to leave a numerator in factored form to help determine whether the expression can be simplified.
b. Multiply the numerator and denominator by the conjugate of $\sqrt{5} - \sqrt{3}$.

\[
\frac{\sqrt{5} + 2}{\sqrt{5} - \sqrt{3}} = \frac{(\sqrt{5} + 2)(\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})} = \frac{\sqrt{5}\sqrt{5} + \sqrt{5}\sqrt{3} + 2\sqrt{5} + 2\sqrt{3}}{(\sqrt{5})^2 - (\sqrt{3})^2} = \frac{\sqrt{30} + \sqrt{18} + 2\sqrt{5} + 2\sqrt{3}}{5 - 3} = \frac{\sqrt{30} + 3\sqrt{2} + 2\sqrt{5} + 2\sqrt{3}}{2}
\]

c. Multiply by the conjugate of $3\sqrt{x} + \sqrt{m}$ to eliminate the radicals from the denominator.

\[
\frac{2\sqrt{m}}{3\sqrt{x} + \sqrt{m}} = \frac{2\sqrt{m}(3\sqrt{x} - \sqrt{m})}{(3\sqrt{x} + \sqrt{m})(3\sqrt{x} - \sqrt{m})} = \frac{6\sqrt{mx} - 2m}{(3\sqrt{x})^2 - (\sqrt{m})^2} = \frac{6\sqrt{mx} - 2m}{9x - m}
\]

PRACTICE

4. Rationalize the denominator.

a. \[
\frac{5}{3\sqrt{5} + 2}
\]
b. \[
\frac{\sqrt{2} + 5}{\sqrt{3} - \sqrt{5}}
\]
c. \[
\frac{3\sqrt{x}}{2\sqrt{x} + \sqrt{y}}
\]

OBJECTIVE

3. Rationalizing Numerators

As mentioned earlier, it is also often helpful to write an expression such as $\frac{\sqrt{3}}{\sqrt{2}}$ as an equivalent expression without a radical in the numerator. This process is called rationalizing the numerator. To rationalize the numerator of $\frac{\sqrt{3}}{\sqrt{2}}$, we multiply the numerator and the denominator by $\sqrt{3}$.

\[
\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3} \cdot \sqrt{3}}{\sqrt{2} \cdot \sqrt{3}} = \frac{\sqrt{9}}{\sqrt{6}} = \frac{3}{\sqrt{6}}
\]

EXAMPLE 5 Rationalize the numerator of $\frac{\sqrt{7}}{\sqrt{45}}$.

Solution

First we simplify $\sqrt{45}$.

\[
\frac{\sqrt{7}}{\sqrt{45}} = \frac{\sqrt{7}}{\sqrt{9 \cdot 5}} = \frac{\sqrt{7}}{3\sqrt{5}}
\]

Next we rationalize the numerator by multiplying the numerator and the denominator by $\sqrt{7}$.

\[
\frac{\sqrt{7}}{3\sqrt{5}} = \frac{\sqrt{7} \cdot \sqrt{7}}{3\sqrt{5} \cdot \sqrt{7}} = \frac{7}{3\sqrt{35}}
\]

PRACTICE

5. Rationalize the numerator of $\frac{\sqrt{32}}{\sqrt{80}}$. 

\[
\frac{\sqrt{32}}{\sqrt{80}} = \frac{\sqrt{32} \cdot \sqrt{80}}{\sqrt{80} \cdot \sqrt{80}} = \frac{\sqrt{2560}}{80} = \frac{16\sqrt{2}}{80} = \frac{\sqrt{2}}{5}
\]
EXAMPLE 6  Rationalize the numerator of $\frac{\sqrt[3]{2x}}{\sqrt[5]{y}}$.

**Solution**  The numerator and the denominator of this expression are already simplified.

To rationalize the numerator, $\sqrt[3]{2x}$, we multiply the numerator and denominator by a factor that will make the radicand a perfect cube. If we multiply $\sqrt[3]{2x}$ by $\sqrt[4]{x}$, we get $\sqrt[3]{8x^3} = 2x$.

$$\frac{\sqrt[3]{2x}}{\sqrt[5]{y}} = \frac{\sqrt[3]{2x} \cdot \sqrt[4]{x}}{\sqrt[5]{y} \cdot \sqrt[4]{x}} = \frac{\sqrt[3]{8x^3}}{\sqrt[20]{20xy}} = \frac{2x}{\sqrt[20]{20xy}}$$

EXAMPLE 7  Rationalize the numerator of $\frac{\sqrt[3]{x} + 2}{5}$.

**Solution**  We multiply the numerator and the denominator by the conjugate of the numerator, $\sqrt[3]{x} - 2$.

$$\frac{\sqrt[3]{x} + 2}{5} \cdot \frac{\sqrt[3]{x} - 2}{\sqrt[3]{x} - 2} = \frac{(\sqrt[3]{x} + 2)(\sqrt[3]{x} - 2)}{5(\sqrt[3]{x} - 2)} = \frac{(\sqrt[3]{x})^2 - 2^2}{5(\sqrt[3]{x} - 2)} = \frac{x - 4}{5(\sqrt[3]{x} - 2)}$$

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

- rationalizing the numerator
- conjugate $\sqrt[3]{3}$
- rationalizing the denominator $\frac{5}{\sqrt[3]{3}}$

1. The ___________ of $a + b$ is $a - b$.
2. The process of writing an equivalent expression, but without a radical in the denominator is, called ___________.
3. The process of writing an equivalent expression, but without a radical in the numerator, is called ___________.
4. To rationalize the denominator of $\frac{5}{\sqrt[3]{3}}$, we multiply by ___________.

Martin-Gay Interactive Videos  Watch the section lecture video and answer the following questions.

5. From Examples 1–3, what is the goal of rationalizing a denominator?
6. From Example 4, why will multiplying a denominator by its conjugate always rationalize the denominator?
7. From Example 5, is the process of rationalizing a numerator any different from rationalizing a denominator?
7.5 Exercise Set

Section 7.5 Rationalizing Denominators and Numerators of Radical Expressions

Rationalize each denominator. See Examples 1 through 3.

1. \( \frac{\sqrt{2}}{\sqrt{7}} \)
2. \( \frac{\sqrt{3}}{2} \)
3. \( \frac{1}{\sqrt{5}} \)
4. \( \frac{1}{\sqrt{2}} \)
5. \( \frac{4}{\sqrt{x}} \)
6. \( \frac{2}{\sqrt{y}} \)
7. \( \frac{4}{\sqrt{3}} \)
8. \( \frac{6}{\sqrt{9}} \)
9. \( \frac{3}{\sqrt{8x}} \)
10. \( \frac{5}{\sqrt{27a}} \)
11. \( \frac{3}{\sqrt{4x^2}} \)
12. \( \frac{5}{\sqrt{3y}} \)
13. \( \frac{9}{\sqrt{3a}} \)
14. \( \frac{x}{\sqrt{5}} \)
15. \( \frac{5}{\sqrt{9}} \)
16. \( \frac{2}{\sqrt{3}} \)
17. \( \frac{2\sqrt{3}}{\sqrt{7}} \)
18. \( \frac{5\sqrt{2}}{\sqrt{11}} \)
19. \( \sqrt{\frac{2x}{5y}} \)
20. \( \sqrt{\frac{13a}{2b}} \)
21. \( \sqrt[3]{\frac{3}{5}} \)
22. \( \sqrt[3]{\frac{7}{10}} \)
23. \( \sqrt{\frac{3y}{50}} \)
24. \( \sqrt{\frac{11y}{45}} \)
25. \( \sqrt{\frac{1}{12z}} \)
26. \( \sqrt{\frac{1}{32x}} \)
27. \( \sqrt{\frac{2}{9x^2}} \)
28. \( \sqrt{\frac{3}{4x^2}} \)
29. \( \sqrt[4]{\frac{81}{8}} \)
30. \( \sqrt[4]{\frac{7}{9}} \)
31. \( \sqrt[4]{\frac{16}{9x^2}} \)
32. \( \sqrt[4]{\frac{32}{m^6n^{13}}} \)
33. \( \frac{5a}{\sqrt[4]{8a^9b^{11}}} \)

Write the conjugate of each expression.

35. \( \sqrt{2} + x \)
36. \( \sqrt{3} + y \)
37. \( 5 - \sqrt{a} \)
38. \( 6 - \sqrt{b} \)
39. \( -7\sqrt{5} + 8\sqrt{x} \)
40. \( -9\sqrt{2} - 6\sqrt{y} \)

Rationalize each denominator. See Example 4.

41. \( \frac{6}{2 - \sqrt{7}} \)
42. \( \frac{3}{\sqrt{7} - 4} \)
43. \( \frac{-7}{\sqrt{x} - 3} \)
44. \( \frac{-8}{\sqrt{y} + 4} \)

Rationalize each numerator. See Example 5.

45. \( \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} + \sqrt{3}} \)
46. \( \frac{\sqrt{3} + \sqrt{4}}{\sqrt{2} - \sqrt{3}} \)
47. \( \frac{\sqrt{a} + 1}{2\sqrt{a} - \sqrt{b}} \)
48. \( \frac{2\sqrt{a} - 3}{2\sqrt{a} + \sqrt{b}} \)
49. \( \frac{8}{1 + \sqrt{10}} \)
50. \( \frac{-3}{\sqrt{6} - 2} \)
51. \( \frac{\sqrt{x}}{\sqrt{x} + \sqrt{y}} \)
52. \( \frac{2\sqrt{a}}{2\sqrt{x} - \sqrt{y}} \)
53. \( \frac{2\sqrt{3} + \sqrt{6}}{4\sqrt{3} - \sqrt{6}} \)
54. \( \frac{4\sqrt{5} + \sqrt{2}}{2\sqrt{5} - \sqrt{2}} \)

Rationalize each numerator. See Example 6.

55. \( \frac{\sqrt[3]{5}}{3} \)
56. \( \frac{\sqrt[3]{3}}{2} \)
57. \( \frac{\sqrt[5]{18}}{5} \)
58. \( \frac{\sqrt[5]{12}}{7} \)
59. \( \frac{\sqrt[4]{4x}}{7} \)
60. \( \frac{\sqrt[4]{3x^3}}{6} \)
61. \( \frac{\sqrt[5]{5y^2}}{\sqrt[5]{4x}} \)
62. \( \frac{\sqrt[4]{9x^4}}{\sqrt[4]{z^3}} \)
63. \( \frac{\sqrt[5]{7}}{5} \)
64. \( \frac{\sqrt[3]{3}}{7} \)
65. \( \frac{\sqrt[2]{2x}}{11} \)
66. \( \frac{\sqrt[2]{y}}{7} \)
67. \( \frac{\sqrt[10]{7}}{8} \)
68. \( \frac{\sqrt[10]{25}}{2} \)
69. \( \frac{\sqrt[10]{3x^5}}{10} \)
70. \( \frac{\sqrt[10]{9y}}{7} \)
71. \( \frac{\sqrt[10]{18x^4y^6}}{3z} \)
72. \( \frac{\sqrt[10]{8x^5y^2}}{2z} \)

Rationalize each numerator. See Example 7.

73. \( \frac{2 - \sqrt{11}}{2} \)
74. \( \frac{\sqrt{15} + 1}{6} \)
75. \( \frac{2 - \sqrt{7}}{-5} \)
76. \( \frac{\sqrt{5} + 2}{\sqrt{2}} \)
77. \( \frac{\sqrt{x} + 3}{\sqrt{x}} \)
78. \( \frac{5 + \sqrt{2}}{\sqrt{2x}} \)
79. \( \frac{\sqrt{2} - 1}{\sqrt{2} + 1} \)
80. \( \frac{\sqrt{8} - \sqrt{3}}{\sqrt{2} + \sqrt{3}} \)
81. \( \frac{\sqrt{x} + 1}{\sqrt{x} - 1} \)
82. \( \frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}} \)
CHAPTER 7  Rational Exponents, Radicals, and Complex Numbers

REVIEW AND PREVIEW

Solve each equation. See Sections 2.1 and 5.8.
83. \(2x - 7 = 3(x - 4)\)  
84. \(9x - 4 = 7(x - 2)\)
85. \((x - 6)(2x + 1) = 0\)  
86. \((y + 2)(5y + 4) = 0\)
87. \(x^2 - 8x = -12\)  
88. \(x^3 = x\)

CONCEPT EXTENSIONS

89. The formula of the radius \(r\) of a sphere with surface area \(A\) is
\[r = \sqrt{\frac{A}{4\pi}}\]
Rationalize the denominator of the radical expression in this formula.

90. The formula for the radius \(r\) of a cone with height 7 centimeters and volume \(V\) is
\[r = \sqrt{\frac{3V}{7\pi}}\]
Rationalize the numerator of the radical expression in this formula.

91. Given \(\frac{\sqrt{5y^3}}{\sqrt{12x^3}}\), rationalize the denominator by following parts (a) and (b).
   a. Multiply the numerator and denominator by \(\sqrt{12x^3}\).
   b. Multiply the numerator and denominator by \(\sqrt{3x}\).
   c. What can you conclude from parts (a) and (b)?

92. Given \(\frac{\sqrt{5y}}{\sqrt{4}}\), rationalize the denominator by following parts (a) and (b).
   a. Multiply the numerator and denominator by \(\sqrt{16}\).
   b. Multiply the numerator and denominator by \(\sqrt{2}\).
   c. What can you conclude from parts (a) and (b)?

Determine the smallest number both the numerator and denominator should be multiplied by to rationalize the denominator of the radical expression. See the Concept Check in this section.
93. \(\frac{9}{\sqrt{5}}\)
94. \(\frac{5}{\sqrt{27}}\)

95. When rationalizing the denominator of \(\sqrt{\frac{5}{7}}\), explain why both the numerator and the denominator must be multiplied by \(\sqrt{7}\).

96. When rationalizing the numerator of \(\sqrt{\frac{5}{7}}\), explain why both the numerator and the denominator must be multiplied by \(\sqrt{5}\).

97. Explain why rationalizing the denominator does not change the value of the original expression.

98. Explain why rationalizing the numerator does not change the value of the original expression.

Integrated Review  RADICALS AND RATIONAL EXPONENTS

Sections 7.1–7.5
Throughout this review, assume that all variables represent positive real numbers.

Find each root.
1. \(\sqrt{81}\)
2. \(\sqrt[2]{-8}\)
3. \(\frac{\sqrt{1}}{\sqrt{16}}\)
4. \(\sqrt{x^6}\)

5. \(\sqrt[5]{y^9}\)
6. \(\sqrt[4]{4y^{10}}\)
7. \(\sqrt{-32y^5}\)
8. \(\sqrt[3]{81b^{12}}\)

Use radical notation to write each expression. Simplify if possible.
9. \(36^{1/2}\)
10. \((3y)^{1/4}\)
11. \(64^{-2/3}\)
12. \((x + 1)^{3/5}\)

Use the properties of exponents to simplify each expression. Write with positive exponents.
13. \(y^{-1/6} \cdot y^{7/6}\)
14. \((2x^{1/3})^4\)
15. \(\frac{x^{1/4} \cdot x^{3/4}}{x^{-1/4}}\)
16. \(4^{1/3} \cdot 4^{2/5}\)

Use rational exponents to simplify each radical.
17. \(\sqrt[3]{8x^6}\)
18. \(\sqrt[5]{a^9b^6}\)
Use rational exponents to write each as a single radical expression.

19. \( \sqrt{x} \cdot \sqrt{x} \)

20. \( \sqrt{5} \cdot \sqrt{2} \)

Simplify.

21. \( \sqrt{40} \)

22. \( \sqrt[4]{16x^7y^{10}} \)

23. \( \sqrt[4]{54x^4} \)

24. \( \sqrt[10]{-64b^{10}} \)

Multiply or divide. Then simplify if possible.

25. \( \sqrt[2]{20} \cdot \sqrt[2]{x} \)

26. \( \sqrt[8]{8x} \cdot \sqrt[8]{8x^2} \)

27. \( \frac{\sqrt{98y^6}}{\sqrt[4]{2y}} \)

28. \( \frac{\sqrt[3]{48a^9b^3}}{\sqrt[3]{ab^3}} \)

Perform each indicated operation.

29. \( \sqrt{20} - \sqrt{75} + 5\sqrt{7} \)

30. \( \sqrt[4]{54y^4} - y\sqrt[4]{16y} \)

31. \( \sqrt[3]{(\sqrt[2]{5} - \sqrt[2]{2})} \)

32. \( (\sqrt[3]{7} + \sqrt[3]{3})^2 \)

33. \( (2x - \sqrt[2]{5})(2x + \sqrt[2]{5}) \)

34. \( (\sqrt[2]{x} + 1 - 1)^2 \)

Rationalize each denominator.

35. \( \frac{\sqrt[3]{7}}{3} \)

36. \( \frac{5}{\sqrt[2]{2x^2}} \)

37. \( \frac{\sqrt[3]{3} - \sqrt[3]{7}}{2\sqrt[3]{3} + \sqrt[3]{7}} \)

Rationalize each numerator.

38. \( \frac{\sqrt[3]{7}}{3} \)

39. \( \frac{\sqrt[3]{9y}}{\sqrt[11]{11}} \)

40. \( \frac{\sqrt[2]{x} - 2}{\sqrt[2]{x}} \)

### 7.6 Radical Equations and Problem Solving

**OBJECTIVES**

1. Solve Equations That Contain Radical Expressions.
2. Use the Pythagorean Theorem to Model Problems.

**OBJECTIVE 1 Solving Equations That Contain Radical Expressions**

In this section, we present techniques to solve equations containing radical expressions such as

\[ \sqrt{2x - 3} = 9 \]

We use the power rule to help us solve these radical equations.

**Power Rule**

If both sides of an equation are raised to the same power, **all** solutions of the original equation are **among** the solutions of the new equation.

This property **does not** say that raising both sides of an equation to a power yields an equivalent equation. A solution of the new equation may or may not be a solution of the original equation. For example, \((-2)^2 = 2^2\), but \(-2 \neq 2\). Thus, each **solution of the new equation must be checked** to make sure it is a solution of the original equation. Recall that a proposed solution that is not a solution of the original equation is called an **extraneous solution**.

**EXAMPLE 1** Solve: \( \sqrt{2x - 3} = 9 \).

**Solution** We use the power rule to square both sides of the equation to eliminate the radical.

\[
\sqrt{2x - 3} = 9 \\
(\sqrt{2x - 3})^2 = 9^2 \\
2x - 3 = 81 \\
2x = 84 \\
x = 42
\]

Now we check the solution in the original equation.

(Continued on next page)